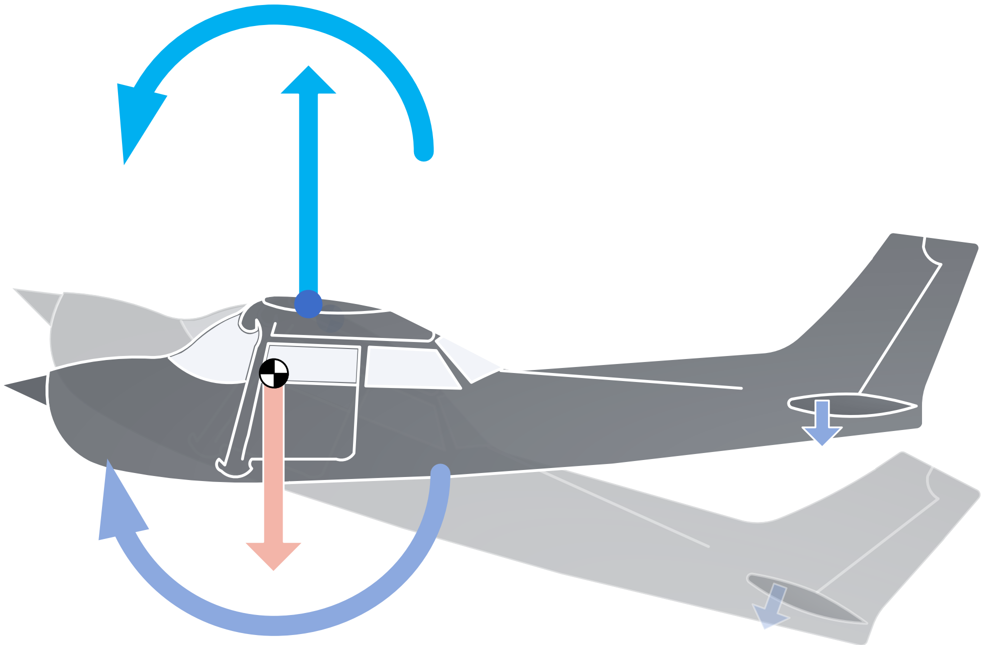
Fakultet prometnih znanosti

Zbirka riješenih zadataka iz

TEORIJE LETA II

Davor Franjković

Karolina Krajček Nikolić



Sveučilište u Zagrebu
Fakultet prometnih znanosti
Zavod za aeronautiku

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Zbirka riješenih zadataka iz TEORIJE LETA II

Zagreb, 2015.

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PREDGOVOR

Ova zbirka zadataka namijenjena je studentima druge godine preddiplomskog studija aeronautike na Fakultetu prometnih znanosti u Zagrebu. Zbirka u potpunosti pokriva nastavni plan i program kolegija Teorija leta II kojeg studenti aeronautike (vojni i civilni piloti, te kontrolori leta) slušaju u trećem semestru. Poglavlja u zbirci prate nastavni plan i program s obzirom na redoslijed i opseg gradiva u okviru nastavnog procesa. Zadaci su ilustrirani crtežima radi lakšeg razumijevanja problematike.

S obzirom da studenti aeronautike nemaju kolegij koji samostalno obrađuje kinematiku i dinamiku općenito, Zbirka u prva tri poglavlja sadrži niz zadataka iz osnova kinematike, dinamike i teorije mehanizama koje su nužne za razumijevanje temeljnih pojmova mehanike leta potrebnih studentu za uspješno praćenje i svladavanje gradiva.

Na kraju Zbirke dana su tri priloga, Karakteristike zrakoplova Cessna Skylane i Cessna Citation 3, Tablica standardne atmosfere i Popis formula kojima se studenti mogu služiti na pismenom dijelu ispita.

Zagreb, srpanj 2015. Autori

SADRŽAJ

1. KINEMATIKA

1.1. Točka se giba u ravnini tako da se zavisnost njezinih koordinata o vremenu može izraziti jednadžbama:

$$\boldsymbol{x = 10 \cdot t}$$

$$\boldsymbol{y = 10 + 8 \cdot t}$$

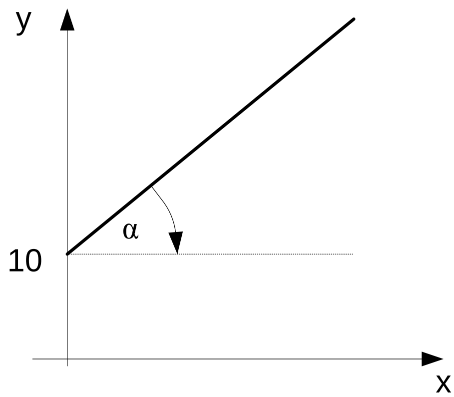
Treba odrediti oblik putanje, brzinu i ubrzanje?

Rješenje:

$$x = 10 \cdot t \quad \Longrightarrow t = \frac{x}{10}$$

$$y = 10 + 8 \cdot \frac{x}{10} = 0.8x + 10$$ ...točka se giba po pravcu

$$\tan \alpha = 0.8 \quad \rightarrow \quad \alpha = \tan^{-1} 0.8 = 38.7°$$

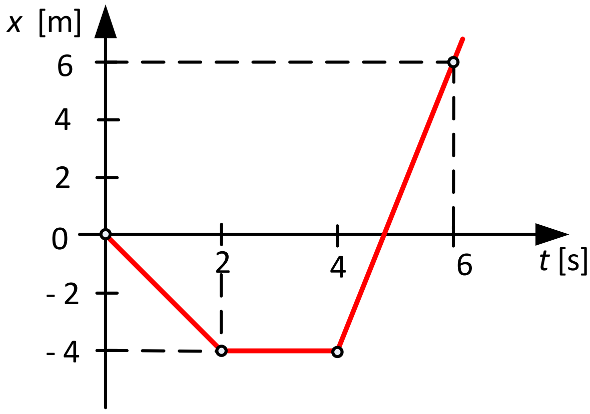


$$v_x = \dot{x} = \frac{dx}{dt} = 10 \qquad a_x = \ddot{x} = \frac{d^2x}{dt^2} = 0$$

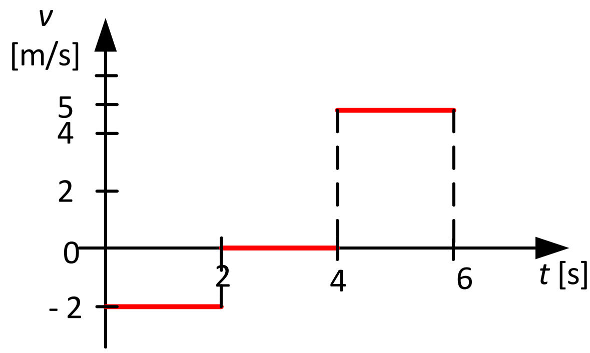
$$v_y = \dot{y} = \frac{dy}{dt} = 8 \qquad a_y = \ddot{y} = \frac{d^2y}{dt^2} = 0$$

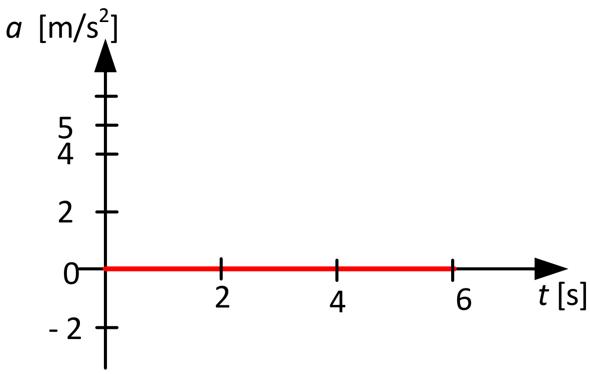
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{10^2 + 8^2} = 12.8\ \frac{\mathrm{m}}{\mathrm{s}} \qquad a = \sqrt{a_x^2 + a_y^2} = 0$$

1.2. Nacrtaj dijagram brzine i dijagram ubrzanja ako je poznat dijagram puta $x(t)$.



Rješenje:





1.3. Odredi brzinu čestice čiji je položaj zadan jednadžbom $\boldsymbol{x = -t^3 + 3t}$.

a) Na kojoj udaljenosti se nalazi čestica u trenutku *t* = 2 s?

b) Kolika joj je pri tome brzina, *t* = 2 s?

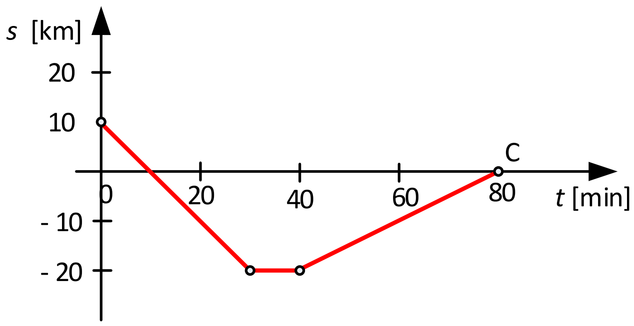
Rješenje:

$$V = \frac{dx}{dt} = -3t^2 + 3$$

a) $x = -t^3 + 3t = -2^3 + 3 \cdot 2 = -8 + 6 = -2$ m

b) $V|_{t=2} = -3 \cdot 2^2 + 3 = -9$ m/s

1.4. Odredite za *s*(*t*) dijagram slijedeće vrijednosti:



a) položaj u trenutku *t* = 0 min

b) položaj u trenutku *t* = 30 min

c) brzinu u trenutku *t* = 20 min

d) brzinu u trenutku *t* = 50 min

e) ubrzanje u trenutku *t* = 20 min

f) Kada bi ovo bio *v*(*t*) dijagram i *s* = 0 km pri *t* = 0 min, koji bi bio položaj u trenutku *t* = 80 min

Rješenje:

a) *s* = 10 km

b) *s* = -20 km

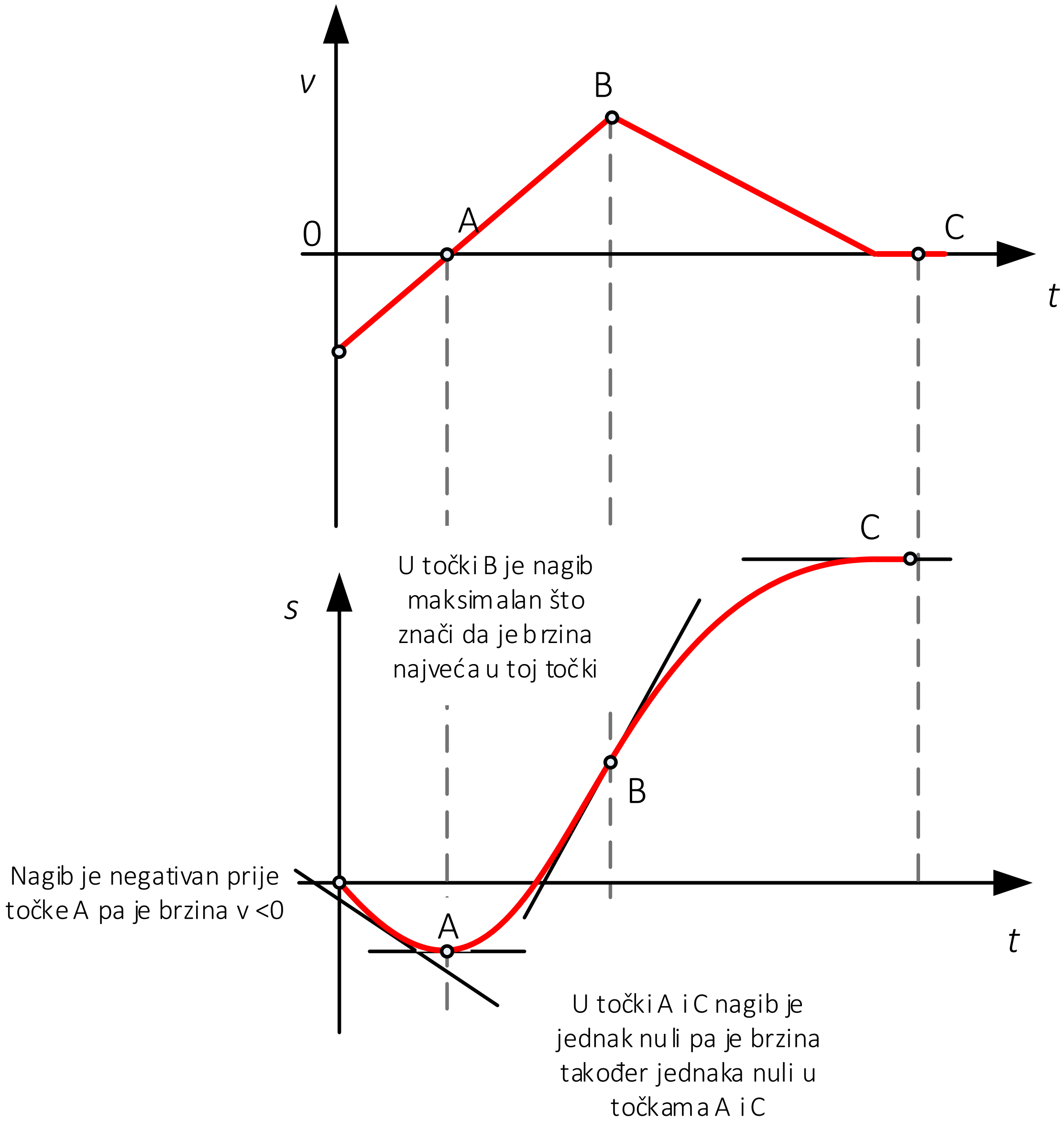
c) $v|_{t\,=\,20\text{ min}} = ds/dt$ =-1 km/min

d) $v|_{t\,=\,50\text{ min}} = ds/dt$ =0.5 km/min

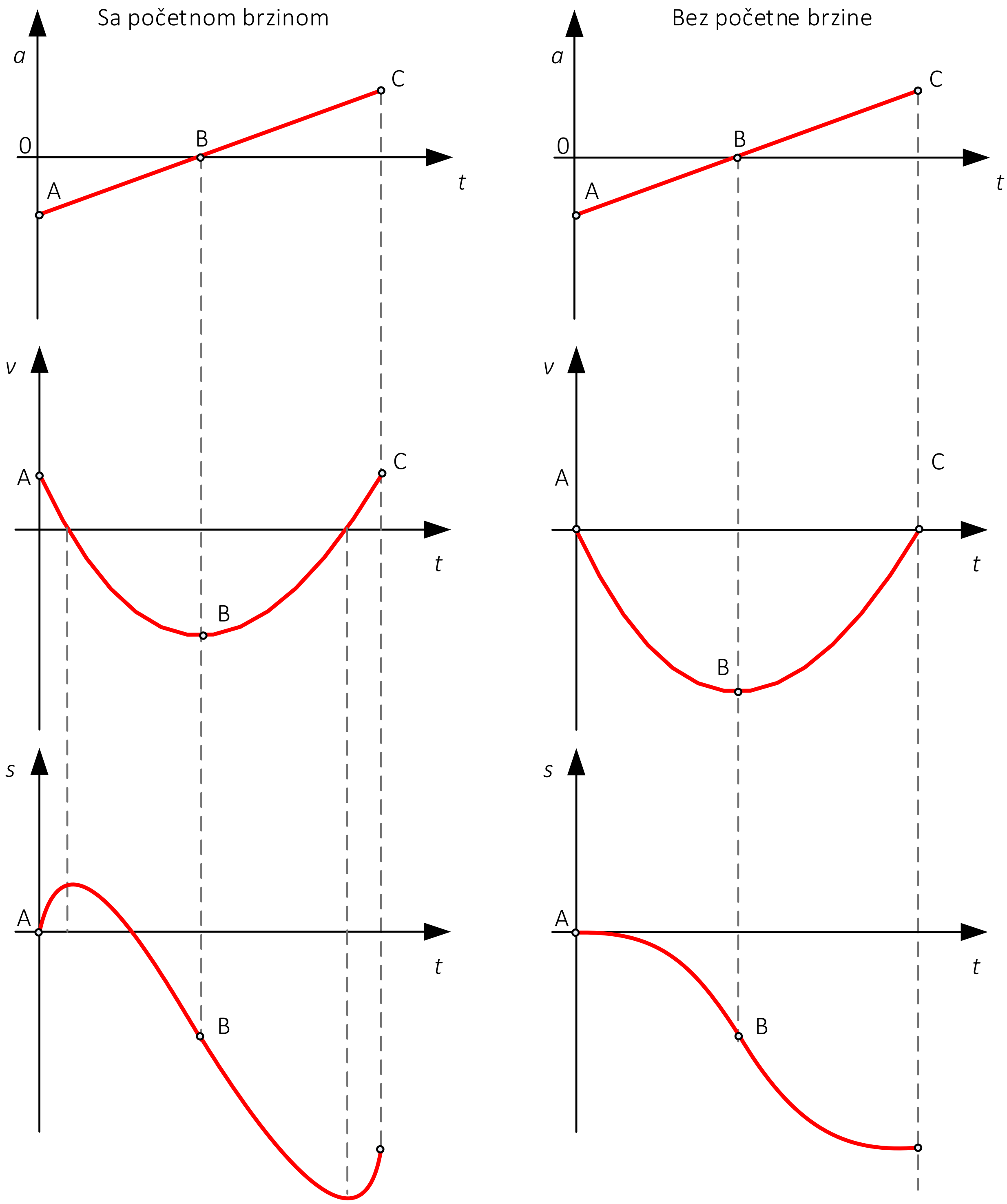
e) ubrzanje u trenutku *t* = 20 min je *a* = 0 km/min^2

f) Položaj u trenutku *t* = 80 min, bio bi -750 km.

1.5. Prema dijagramu brzine $v = f(t)$ odredite dijagram puta $s(t)$.



1.6. Odredite dijagrame puta i brzine prema dijagramu zavisnosti ubrzanja o vremenu prikazanom na slici.



Točku infleksije određujemo prema drugoj derivaciji funkcije. S obzirom da je druga derivacija po putu jednaka ubrzanju a, slijedi kako je za $a < 0$ f-cija konveksna, a za $a > 0$ konkavna.

1.7. Promatra se let aviona kao krutog tijela, što znači da se njegovo gibanje poistovjećuje s gibanjem njegovog težišta. Gibanje aviona odvija se u koordinatnom sustavu 0xyz vezanom za točku na površini zemlje, i to samo u vertikalnoj ravnini *xz*. U određenom trenutku avion se nalazi na x_0 = 0 m i z_0 = 5000 m i leti horizontalno brzinom $\vec{v}_0 = 100 \cdot \vec{i}$ [m/s]. Pilot tada započinje prijelazno gibanje koje traje *t* = 20 s, pri čemu je ubrzanje aviona jednako $\vec{a} = \left(-0{,}05 \cdot t^2 + t\right) \cdot \vec{i} + \left(0{,}4 \cdot t - 4\right) \cdot \vec{k}$.

a) Napiši komponente ubrzanja a_x i a_z. Nacrtaj dijagrame a_x = f(*t*) i a_z = f(*t*). Odredi veličinu ubrzanja *a* = f(*t*) i izračunaj za t = 20 s.

b) Odredi komponente brzine v_x i v_z. Nacrtaj dijagrame v_x = f(*t*) i v_z = f(*t*). Odredi veličinu brzine = f(*t*) i izračunaj za t = 20 s.

c) Odredi zavisnost prevaljenog horizontalnog puta o vremenu *x* = f(*t*). Nacrtaj dijagram *x* = f(*t*). Kolika je ukupno prevaljena horizontalna udaljenost?

d) Odredi zavisnost visine leta o vremenu *z* = f(*t*). Nacrtaj dijagram *z* = f(*t*). Na kojoj visini se avion nalazi nakon prijelaznog gibanja od 20 s?

e) Nacrtaj trajektoriju aviona *z* = *z* (*x*) i na njoj označi vremenske intervale.

Napomena: Raditi s najmanje pet vremenskih trenutaka.

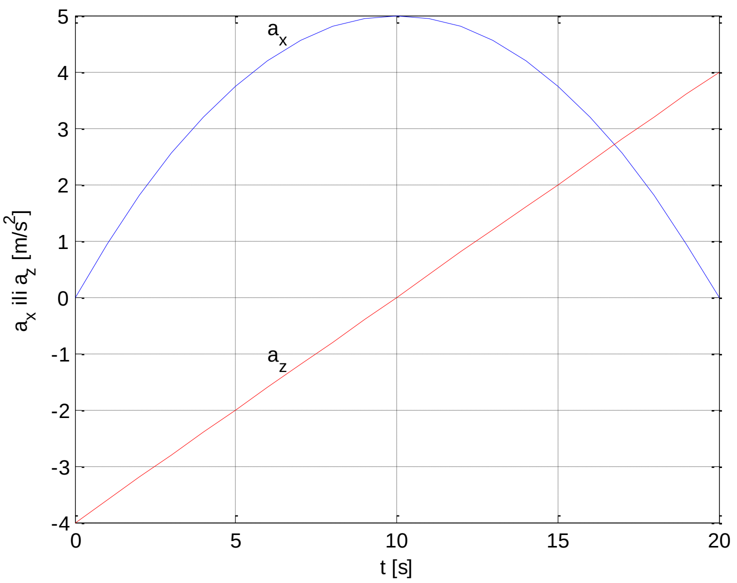
Rješenje:

a) $a_x = -0.05t^2 + t$

$a_z = 0.4 \cdot t - 4$

t [s]	0	5	10	15	20
a_x [m/s²]	0	3.75	5	3.75	0
a_z [m/s²]	-4	-2	0	2	4

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{0^2 + 4^2} = 4 \text{ m/s}^2$$



b)

$$v_x = \int a_x dt = \int (-0.05t^2 + t)dt = -0.05\frac{t^3}{3} + \frac{t^2}{2} + C$$

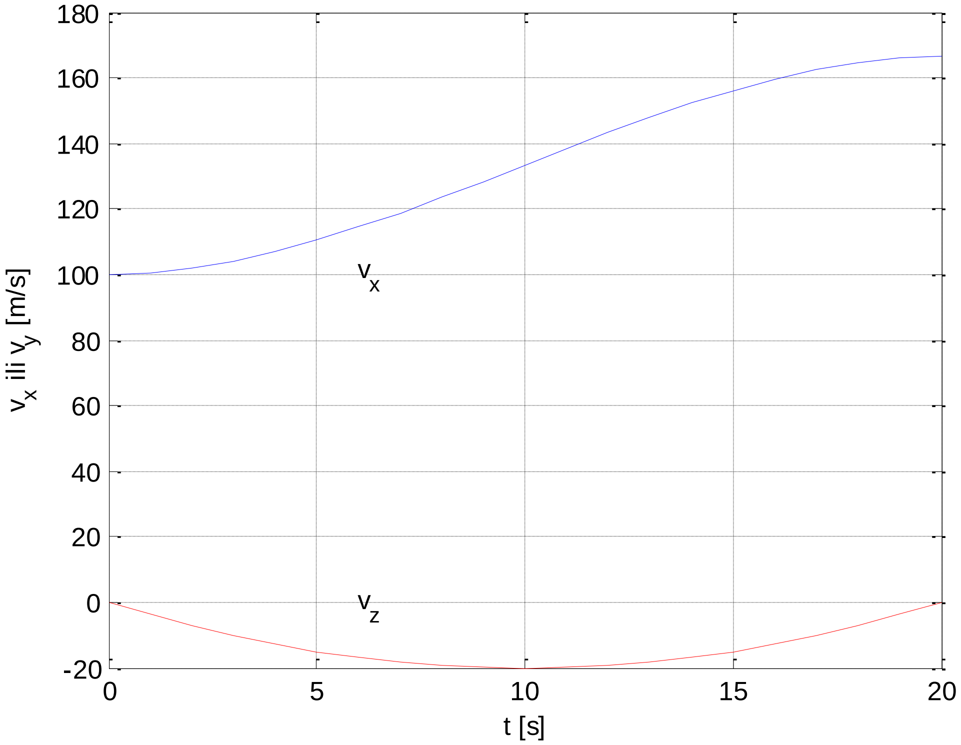
$$v_x = -\frac{5t^3}{300} + \frac{t^2}{2} + 100$$

$$v_z = \int a_z dt = \int (0.4t - 4)dt = 0.4\frac{t^2}{2} - 4t + C$$

$$v_z = 0.2t^2 - 4t$$

t [s]	0	5	10	15	20
v_x [m/s]	100	110.42	133.33	156.2500	166.67
v_z [m/s]	0	-15	-20	-15	0

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{166.67^2 + 0^2} = 198.24 \text{ m/s}$$

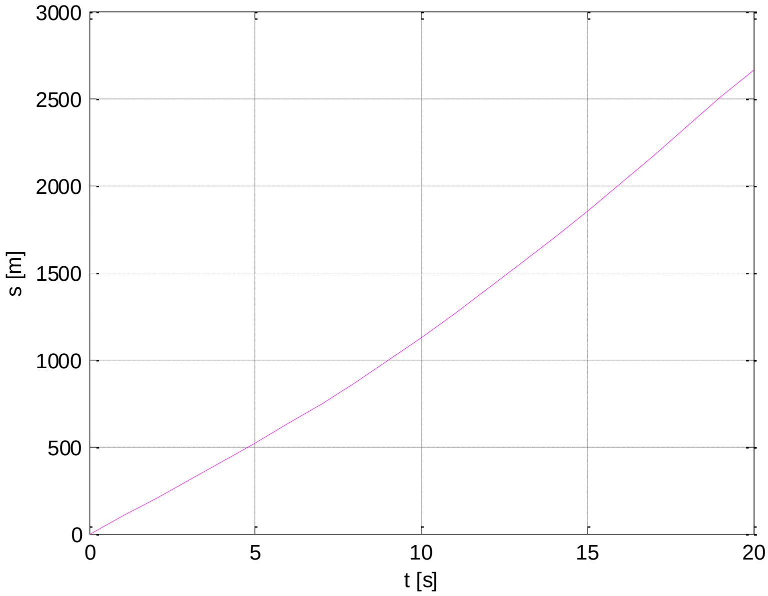


c)

$$x = \int v_x dt = \int \left(-\frac{5t^3}{300} + \frac{t^2}{2} + 100\right) dt = -\frac{5}{300}\frac{t^4}{4} + \frac{t^3}{6} + 100t + C$$

$$x = -\frac{t^4}{240} + \frac{t^3}{6} + 100t$$

t [s]	0	5	10	15	20
x [m]	0	5182	1125	1851.6	2666.7

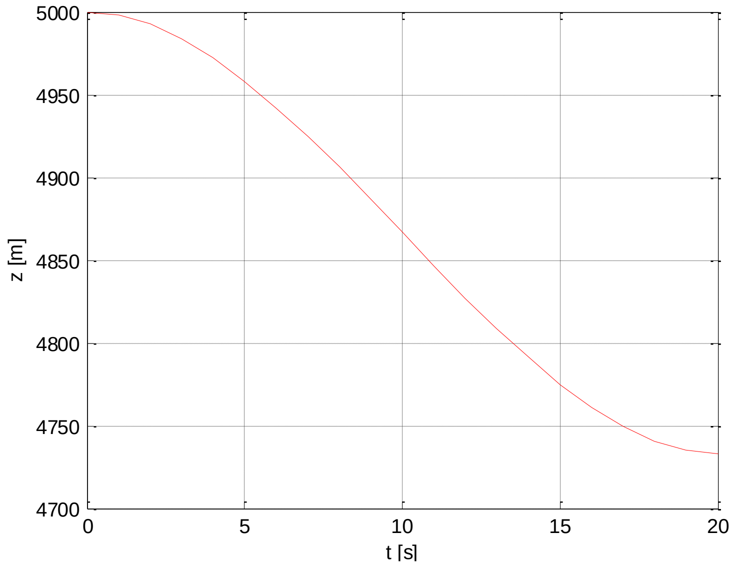


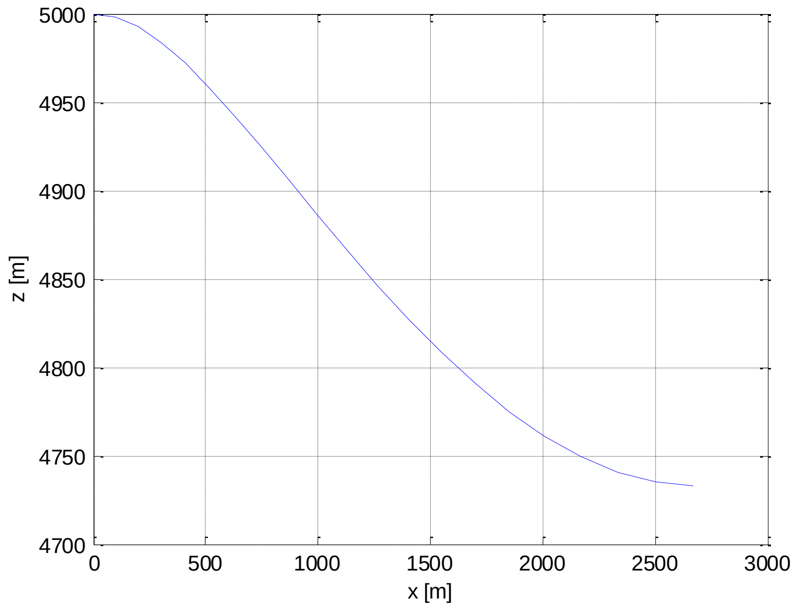
d)

$$z = \int v_z dt = \int (0{,}2t^2 - 4t)dt = \frac{1}{5}\frac{t^3}{3} - \frac{4t^2}{2} + C$$

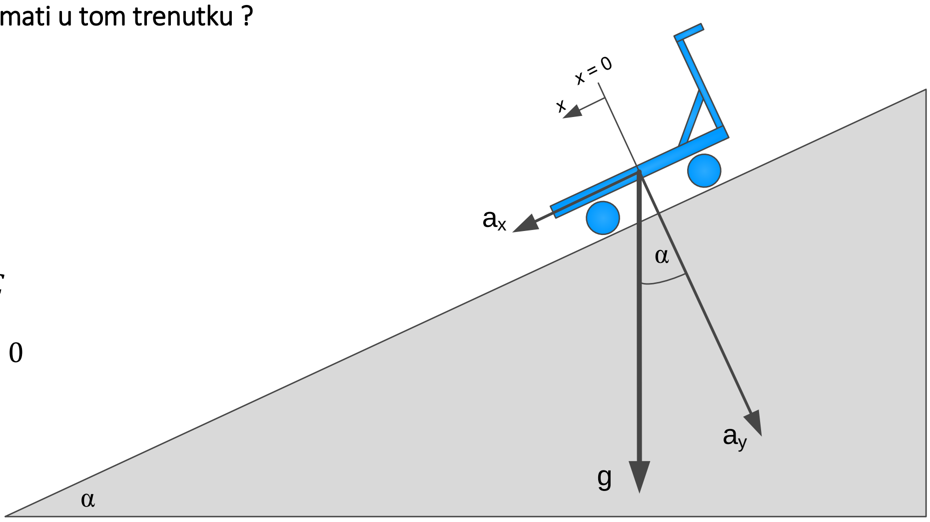
$$z = \frac{t^3}{15} - 2t^2 + 5000$$

t [s]	0	5	10	15	20
z [m]	5000	4958.3	4866.7	4775	4733.3





1.8. Kolica se gibaju iz stanja mirovanja niz kosinu pod kutom od 30°. Koliko vremena treba da prevale put *s* = 2 m ? Koliku će brzinu imati u tom trenutku ?



Rješenje:

$$\dot{x} = v = a_x \cdot t \quad /\int$$

$$x = \int a_x \cdot tdt = \frac{at^2}{2} + C$$

Za $t = 0$ i $x = 0 \ \rightarrow C = 0$

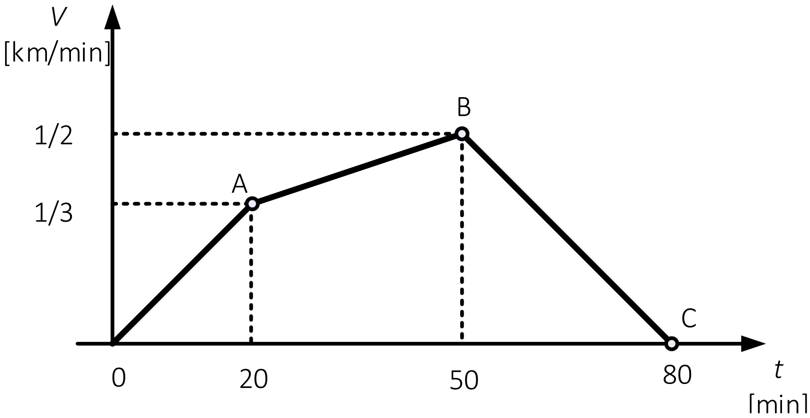
$$\boxed{x = \frac{a_x t^2}{2}}$$

$$\sin\alpha = \frac{a_x}{g} \qquad \Longrightarrow \qquad a_x = g\sin\alpha = 9.81\sin 30° = 4.905 \ \ \mathrm{m/s^2}$$

$$t = \sqrt{\frac{2x}{a_x}} = \sqrt{\frac{2\cdot 2}{4.905}} = 0.815 \ \ \mathrm{s}$$

$$v = a_x \cdot t = 4.905 \cdot 0.815 = 4 \ \ \mathrm{m/s}$$

1.9. Zadan je dijagram brzine gibanja nekog vozila. Koliki je put prevalilo neko vozilo u intervalu $t = 0$ do $t = 80$ min ?



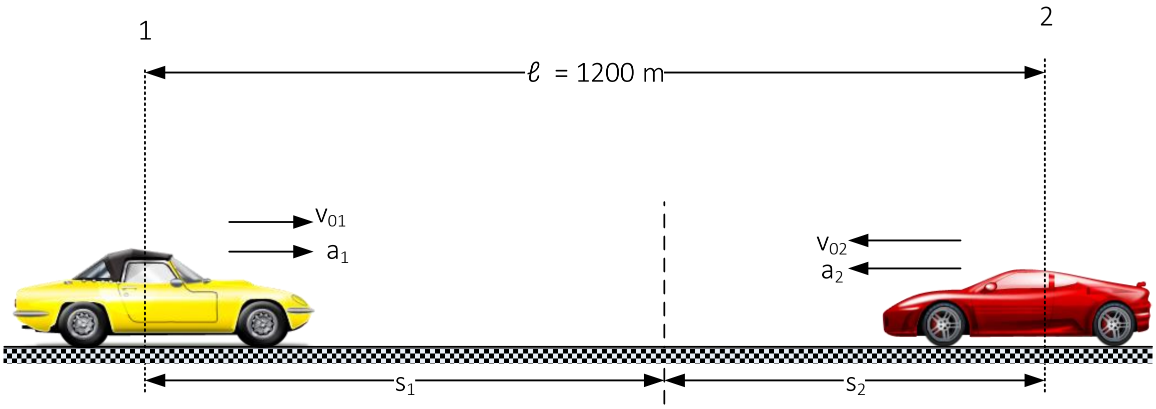
Rješenje:

$$\bar{v} = \frac{1}{2}(v_0 + v)$$

$$s = \bar{v} \cdot t = \frac{1}{2}(v_0 + v) \cdot t$$

$$s = \frac{1}{2} \cdot \frac{1}{3} \cdot 20 + \frac{1}{2}\left(\frac{1}{3} + \frac{1}{2}\right) \cdot 30 + \frac{1}{2}\left(\frac{1}{2} + 0\right) \cdot 30 = 23.35 \ \text{km}$$

1.10. Dva vozila gibaju se jedno prema drugome po pravcu ubrzanjima a_1 = 10 m/s^2 i a_2 = 7 m/s^2 i s početnim brzinama v_{01} = 18 m/s i v_{02} = 12 m/s. Njihova međusobna udaljenost u početnom trenutku je ℓ = 1200 m. Nakon koliko vremena će se vozila mimoići ?



Rješenje:

$$\ell = s_1 + s_2$$

$$\ell = v_{01}t + \frac{a_1 t^2}{2} + v_{02}t + \frac{a_2 t^2}{2}$$

$$1200 = 18t + 5t^2 + 12t + 3.5t^2$$

$$\ddot{x} = a$$

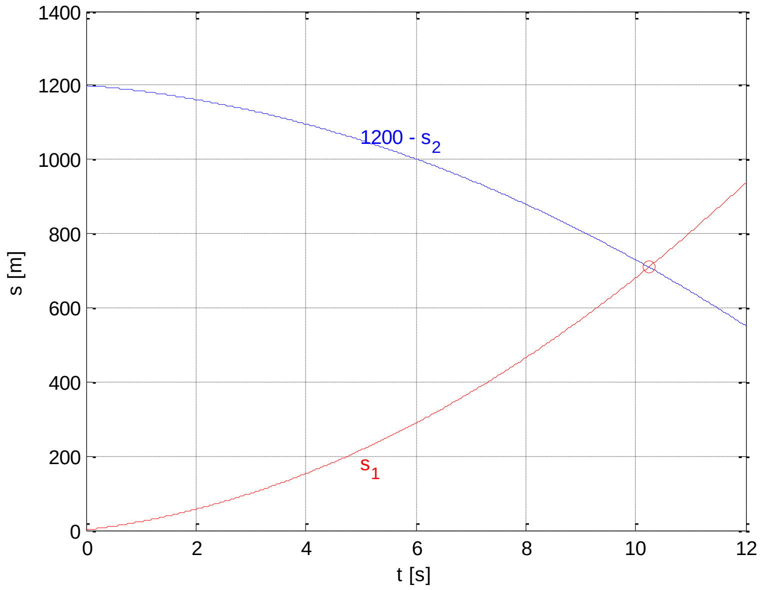
$$v = \int \ddot{x}\, dt = \int a \cdot dt = at + C_1$$

$$s = \int \dot{x}\, dt = \int (at + C_1) dt = \frac{at^2}{2} + C_1 t + C_2$$

$$8.5t^2 + 30t - 1200 = 0$$

$$t_{1,2} = \frac{-30 \pm \sqrt{30^2 + 4 \cdot 8.5 \cdot 1200}}{2 \cdot 8.5} = \frac{-30 \pm 204}{17}$$

$$t = \frac{-30 + 204}{17} = 10.23 \text{ s}$$



Crvena linija na grafu:

$$s_1 = v_{01}t + \frac{a_1 t^2}{2}$$

Plava linija na grafu:

$$\ell - s_2 = \ell - v_{02}t - \frac{a_2 t^2}{2}$$

1.11. Kotač polumjera R = 20 cm počne se okretati kutnim ubrzanjem $\boldsymbol{\varepsilon = 2\pi}$ rad/s^2 u negativnom smjeru (suprotno kretanju kazaljke na satu). Koliki su brzina i ubrzanje točke N poslije vremena t = 5 s od početka kretanja ?

Rješenje:

$$\omega = \varepsilon \cdot t = 2\pi \cdot 5 = 10\pi \quad \text{rad/s}$$

$$v = R \cdot \omega = 0.2 \cdot 10\pi = 2\pi \quad \text{m/s}$$

$$\hat{\varphi} = \frac{1}{2}\varepsilon\, t^2 = \frac{1}{2} 2\pi\, t^2 = 25\pi \quad \text{rad}$$

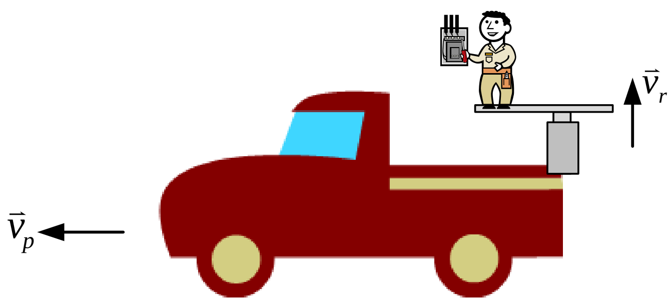
$$a_N = \frac{v^2}{R} = \frac{(R \cdot \omega)^2}{R} = R\,\omega^2$$

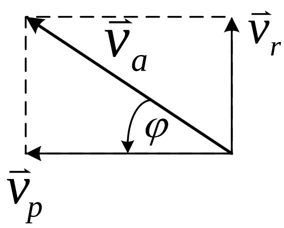
$$a_T = \frac{v}{t} = \frac{R \cdot \omega}{t} = R\,\varepsilon$$

$$a = \sqrt{a_T^2 + a_N^2} = \sqrt{(R\,\varepsilon)^2 + (R\,\omega^2)^2} = \sqrt{R^2(\varepsilon^2 + \omega^4)} = R\sqrt{\varepsilon^2 + \omega^4}$$

$$a = 0.2\sqrt{(2\pi)^2 + (10\pi)^4} = 197.4 \ \text{m/s}^2$$

1.12. Na kamion je učvršćena pokretna dizalica koja služi za održavanje uličnih električnih instalacija. Dizalica se može dizati i spuštati vertikalno naviše i naniže. Pri pomicanju s jednog radnog mjesta na drugo kamion se kreće po ravnom putu konstantnom brzinom v_p = 1.5 m/s, a dizalica vertikalno naviše, konstantnom brzinom v_r = 0.5 m/s. Odrediti veličinu brzine kretanja čovjeka na dizalici i kut $\boldsymbol{\varphi}$ koji njegova putanja zatvara s horizontalnim pravcem kretanja kamiona.





Rješenje:

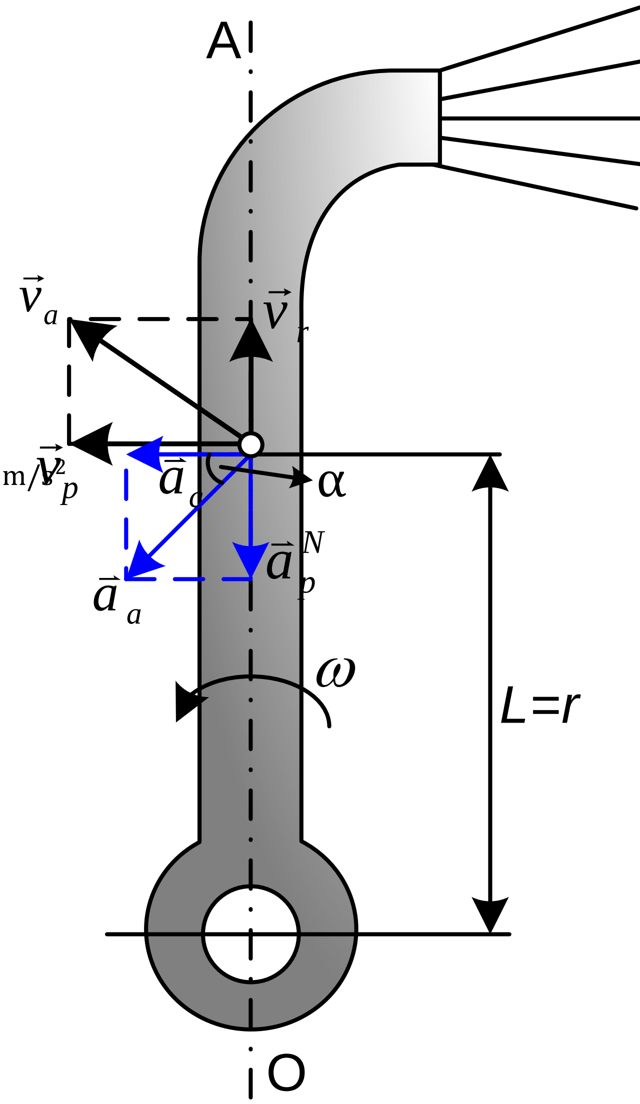
$$\vec{v}_a = \vec{v}_p + \vec{v}_r$$

$$v_a = \sqrt{v_p^2 + v_r^2} = \sqrt{1.5^2 + 0.5^2} = 1.58 \ \text{m/s}$$

$$\text{tg}\,\varphi = \frac{0.5}{1.5} = \frac{1}{3} \qquad \rightarrow \quad \varphi = 18^\circ\, 25'$$

1.13. Iz cijevi $\overline{\boldsymbol{OA}}$ koja se jednoliko okreće u ravnini sa n = 60 o/min oko osi u O, struji mlaz vode. Mlaz struji kroz cijev brzinom v_r = 1.9 m/s u pravcu $\overline{\boldsymbol{OA}}$. Odrediti veličinu, pravac i smjer Coriolisovog ubrzanja a_c? Odrediti apsolutnu brzinu i apsolutno ubrzanje čestice vode, ako se ona nalazi na udaljenosti 80 mm od O ?

Rješenje:



$$\omega = \frac{n\pi}{30} = \frac{60\pi}{30} = 2\pi \ \ \mathrm{s}^{-1}$$

$$a_p^T = 0 \quad (jer\ je\ \omega = konst.)$$

$$v_p = r \cdot \omega = 0.08 \cdot 2\pi = 0.5 \ \ \mathrm{m/s}$$

$$v_a = \sqrt{v_p^2 + v_r^2} = \sqrt{1.9^2 + 0.5^2} = 1.965 \ \ \mathrm{m/s}$$

$$a_c = 2\vec{\omega} \times \vec{v}_r = 2\omega \cdot v_r \cdot \sin\sphericalangle(\vec{\omega}, \vec{v}_r) = 2 \cdot 2\pi \cdot 1.9 \cdot 1 = 24 \ \ \mathrm{m/s^2}$$

$$a_p^N = \frac{v^2}{r} = \frac{(r \cdot \omega)^2}{r} = r\,\omega^2 = 0.08 \cdot (2\pi)^2 = 3.16 \ \ \mathrm{m/s^2}$$

$$a_p = \sqrt{\left(a_p^N\right)^2 + \left(a_p^T\right)^2} = a_p^N = 3.16 \ \ \mathrm{m/s^2}$$

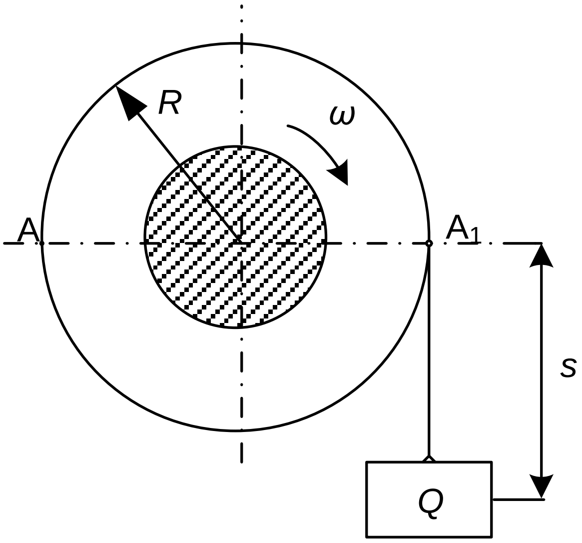
$$a_a = \sqrt{a_p^2 + a_c^2} = \sqrt{24^2 + 3.16^2} = 24.2 \ \ \mathrm{m/s^2}$$

$$tg\,\alpha = \frac{a_p}{a_c} = \frac{3.16}{24} = 0.134 \qquad \rightarrow \qquad \alpha = 7^\circ\,30'$$

Uvijek kada se točka giba po putanji koja ujedno i rotira, javlja se dodatna komponenta ubrzanja. Ta komponenta zove se Coriolisovo ubrzanje, a nastaje kao posljedica spregnutog gibanja točke po putanji i rotacije putanje.

1.14. Teret Q svojom težinom vrši okretanje valjka polumjera *R* = 25 cm prema slici. Gibanje tereta zadano je jednadžbom $\boldsymbol{s = 0.5t^2 - t}$, u kojoj je s udaljenost tereta od horizontale A-A$_1$. Izračunati kutnu brzinu $\boldsymbol{\omega}$ i kutno ubrzanje $\boldsymbol{\varepsilon}$ valjka u trenutku *t* = 6 s, kao i ubrzanje točke na obodu valjka.

Rješenje:



$$s = 0.5t^2 - t \qquad [\mathrm{m}]$$

$$v = t - 1 \qquad [\mathrm{m/s}]$$

$$t = 6 \qquad [\mathrm{s}]$$

$$v = 6 - 1 = 5 \quad \mathrm{m/s}$$

$$\omega = \frac{v}{R} = \frac{5}{0.25} = 20 \ \ \mathrm{s^{-1}}$$

$$a = \frac{dv}{dt} = 1 \ \ \mathrm{m/s^2}$$

$$\varepsilon = \frac{d\omega}{dt} = \frac{1}{R}\frac{dV}{dt} = \frac{a}{R} = \frac{1}{0.25} = 4 \quad \mathrm{s^{-2}}$$

1.15. Čestica se u trenutku *t* = 0 nalazi na udaljenosti s_0 od pola O i ima brzinu v_0. Ako je ubrzanje *a* konstantno, odrediti kinematičke dijagrame pravocrtnog gibanja.

Rješenje:

Integriranjem konstantnog ubrzanja *a* dobiva se brzina, a integriranjem tako dobivene brzine, put čestice:

$$a = \frac{dv}{dt} \quad \Rightarrow \quad dv = adt$$

$$v = \int adt$$

$$v = at + C_1$$

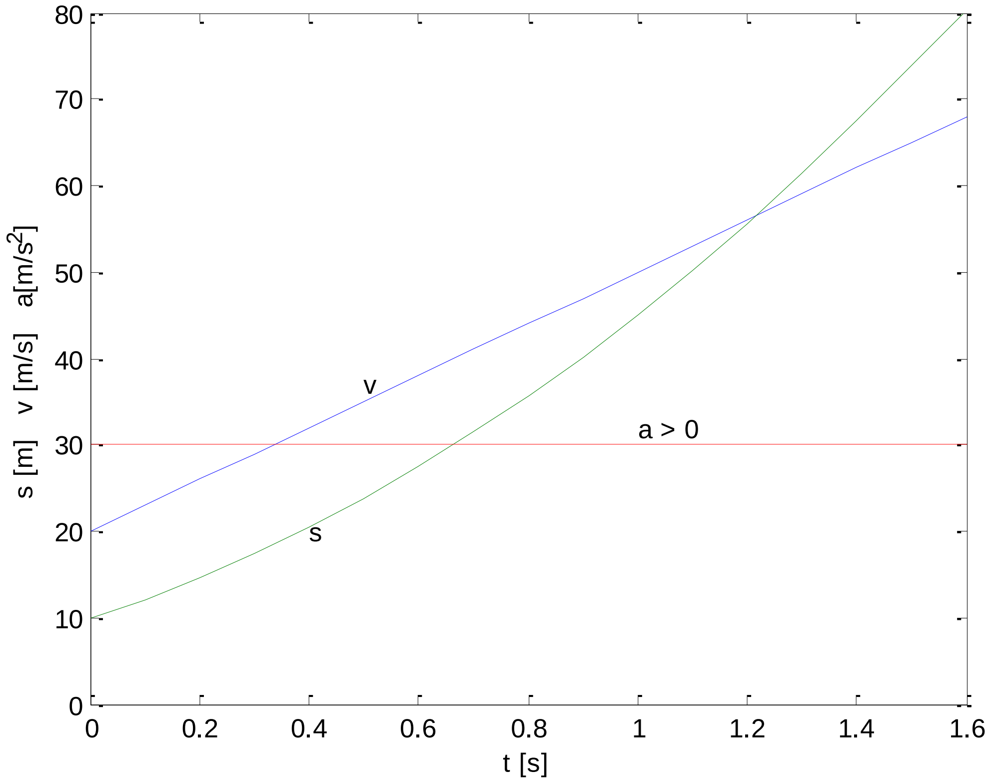
$$s = a\frac{t^2}{2} + C_1 + C_2$$

Budući da je za $t = 0, v = v_0$ *i* $s = s_0$ *konstante su* $C_1 = v_0$ *i* $C_2 = s_0$*, pa je:*

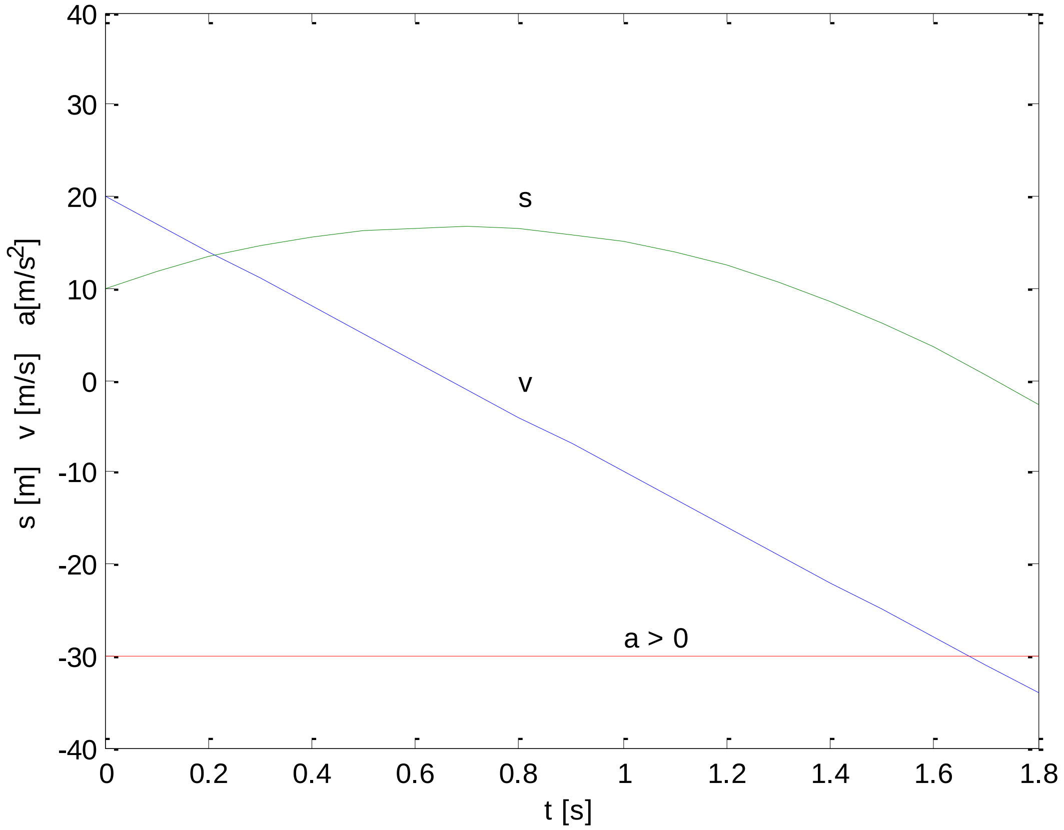
$$v = at + v_0$$

$$s = a\frac{t^2}{2} + v_0 + s_0$$

Ako su s_0 i v_0 pozitivne veličine, te ako je ubrzanje veće od 0, čestica se udaljuje od ishodišta O, brzina se u jednakim vremenskim razmacima povećava za isti iznos, a gibanje je jednoliko ubrzano.



Kad je ubrzanje negativno, brzina u prvom dijelu gibanja se jednoliko smanjuje, pa se u nekom trenutku čestica zaustavlja i nastavlja gibanje u suprotnom smjeru povećavajući brzinu jednoliko.



```
%% Kinematicki dijagrami
v0=20;
s0=10;
a=30;
t=0:0.1:1.8;
v=a*t+v0;
s=a*t.^2/2+v0*t+s0;
plot(t,v,t,s,t,a*ones(1,length(t)))
xlabel('t [s]');ylabel('s [m]   v [m/s]   a[m/s^2]')
axis([0 1.6 0 80])
text(0.4,20,'s')
text(0.5,37,'v')
text(1,32,'a > 0')

figure(2)
v=-a*t+v0;
s=-a*t.^2/2+v0*t+s0;
plot(t,v,t,s,t,-a*ones(1,length(t)))
xlabel('t [s]');ylabel('s [m]   v [m/s]   a[m/s^2]')
axis([0 1.8 -40 40])
text(0.8,20,'s')
text(0.8,0,'v')
text(1,-28,'a < 0')
```

1.16. Cilindrični rotor stroja rotira se u uljnoj kupki tako da se kutno ubrzanje mijenja prema zakonu $\boldsymbol{\varepsilon = -k\omega^2}$ gdje je k konstantna veličina. Mjerenjem je ustanovljeno da se kutna brzina s početnih 300 s^{-1} smanji na polovicu nakon 5 s. Koliki su iznosi brzine i komponenata ubrzanja nakon daljnjih 5 s točke na obodu rotora polumjera 0.5 m?

Rješenje:

Iznosi brzine i komponenata ubrzanja točke određuju se s pomoću izraza:

$$v = R\dot{\varphi} = R\omega$$

$$a_T = R\ddot{\varphi} = R\varepsilon$$

$$a_N = \frac{v^2}{R} = R\omega^2$$

Iz promjene kutnog ubrzanja $\varepsilon = \dot{\omega} = -k\omega^2$ slijedi da je:

$$\frac{d\omega}{dt} = -k\omega^2$$

$$kdt = -\frac{d\omega}{\omega^2}$$

Nakon integriranja dobije se za vrijeme t:

$$\int kdt = -\int \frac{1}{\omega^2} d\omega$$

$$kt = \frac{1}{\omega} + C_1$$

Postavljamo rubne uvjete: za $t = 0\ \mathrm{s}$ → $\omega_0 = 300\ \mathrm{s^{-1}}$ pa je $C_1 = -\frac{1}{300}$

za $t = 5\ s$ → $\omega_5 = 150\ s^{-1}$ *pa je* $k = \frac{1}{750} - \frac{1}{1500} = \frac{1}{1500}$

$$\omega = \frac{1500}{5+t}$$

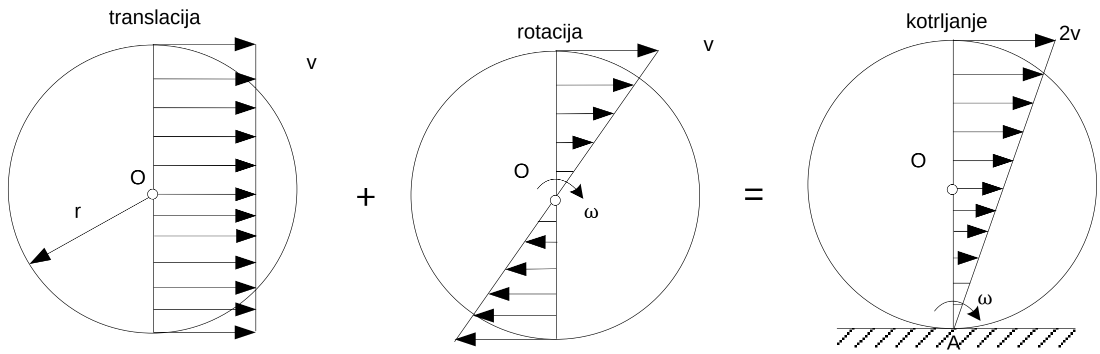
$t = 10\ \mathrm{s}$ → $\omega_{10} = 100\ \mathrm{s^{-1}}$ i $\varepsilon_{10} = -6.67\ \mathrm{s^{-2}}$ ... rotor se usporava

$v = R\omega_{10} = 50\ \ \mathrm{m/s}$

$a_T = R\varepsilon_{10} = -3.33\ \ \mathrm{m/s^2}$

$a_N = R\omega_{10}^2 = 5000\ \mathrm{m/s^2}$

1.17. Lokomotiva vozi brzinom *v* = 140 km/h. Odrediti broj okretaja *n* njezina kotača, promjer kojega je 2*R* = 2000 mm, i kutno ubrzanje pri ubrzavanju vlaka od *a* = 0.2 m/s².



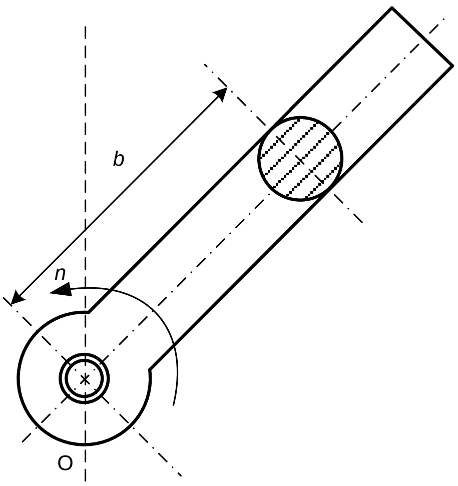
Rješenje:

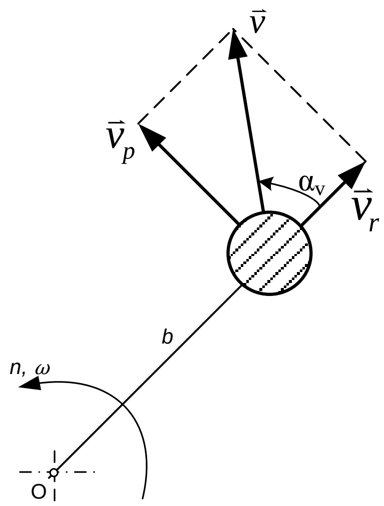
Gibanje kotača može se rastaviti u translaciju (sve točke na kotaču imaju istu brzinu) i rotaciju oko O. Zbroj tih dvaju gibanja daje kotrljanje. Brzina translacije je brzina gibanja lokomotive *v*. Kutna brzina rotacije oko O je $\omega = v/R$. Oba gibanja zbrojena daju kotrljanje s pravocrtnom raspodjelom brzina, što se opet može zamisliti kao trenutna rotacija oko točke A kutnom brzinom $\omega = v/R$. Točka A (trenutni pol brzina) u svakom trenutku ima drugi položaj. Kutno ubrzanje pri kotrljanju $\varepsilon = a/R$, gdje je *a* ubrzanje pri translaciji, a jednako je tangencijalnom ubrzanju rotacije oko O. Uz zadane vrijednosti:

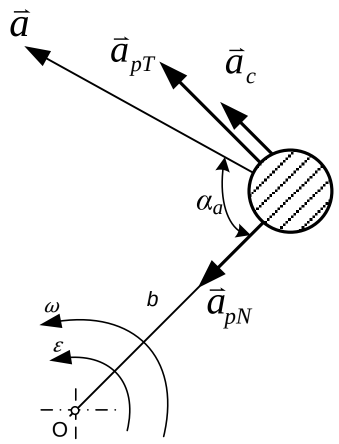
$$\omega = 2\pi n \quad \Rightarrow \quad n = \frac{\omega}{2\pi} = \frac{v/R}{2\pi} = v\frac{R}{2\pi} = \frac{140\,000}{3600}\frac{1}{2\pi} = 6.189\ \mathrm{o/s} = 372\ \mathrm{o/min}$$

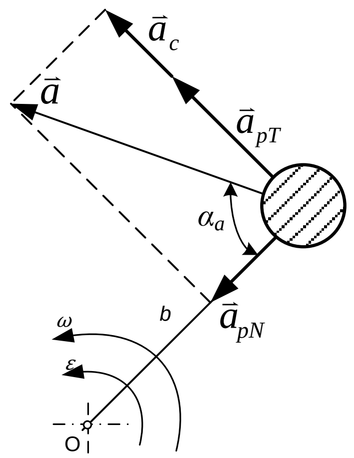
$$\varepsilon = \frac{a}{R} = \frac{0.2}{1} = 0.2\ \mathrm{s^{-2}}$$

1.18. Čestica se giba u cijevi konstantnom brzinom u odnosu na cijev $\boldsymbol{v_r} = \mathbf{5\ m/s}$. Cijev rotira oko O tako da se s početne brzine vrtnje n = 150 min^{-1} ubrzava jednoliko na n_1 = 1000 min^{-1}, za t = 0.1 s. Odrediti vektore apsolutne brzine i ubrzanja na početku razdoblja ubrzavanja kada se čestica nalazila na udaljenosti b = 0.2 m od središta rotacije O.









Rješenje:

Relativno gibanje čestice u odnosu na cijev jednoliko je pravocrtno s brzinom v_r. Prijenosno gibanje za česticu jest rotacija cijevi oko središta O, pa čestica na početku perioda ubrzavanja ima prijenosnu brzinu:

$$v_p = b\omega = b\frac{n\pi}{30} = 0.2\cdot\frac{150\pi}{30} = 3.142\ ms^{-1}$$

Apsolutna brzina iznosi:

$$v = \sqrt{v_p^2 + v_r^2} = \sqrt{3.142^2 + 5^2} = 5.905\ \text{ms}^{-1}$$

Vektor apsolutne brzine leži u odnosu na cijev pod kutom:

$$\alpha_v = arctg\frac{v_p}{v_r} = arctg\frac{3.142}{5} = 32.142\ °$$

Relativno ubrzanje jednako je nuli, jer je relativno gibanje jednoliko pravocrtno. Prijenosna normalna komponenta ubrzanja iznosi (usmjerena prema središtu prijenosne rotacije O):

$$a_{pN} = b\omega^2 = b\left(\frac{n\pi}{30}\right)^2 = 0.2\left(\frac{150\pi}{30}\right)^2 = 49.348\ \text{ms}^{-2}$$

Prijenosno kutno ubrzanje je:

$$\varepsilon = \frac{\omega_1 - \omega}{t} = \frac{(n_1 - n)\pi}{30t} = \frac{(1\,000 - 150)\pi}{30 \cdot 0.1} = 890.118\ \text{s}^{-2}$$

Tangencijalna komponenta prijenosnog ubrzanja:

$$a_{pT} = b\varepsilon = 0.2 \cdot 890.118 = 178.024\ \text{ms}^{-2}$$

Budući da se cijev ubrzava, smjer ε poklapa se sa smjerom ω te to određuje i smjer tangencijalnog ubrzanja $\vec{a}_{pT}$. Pravac tog ubrzanja okomit je na cijev. Vektor $\vec{\omega}$ okomit je na ravninu u kojoj se giba čestica, pa se Coriolisovo ubrzanje $\vec{a}_c = 2\vec{\omega} \times \vec{v_r}$ poklapa se po smjeru s vektorom $\vec{a}_{pT}$. Po iznosu je:

$$a_c = 2\omega v_r \sin 90° = 2 \cdot 15.71 \cdot 5 \sin 90° = 157.080\ \text{ms}^{-2}$$

Apsolutno ubrzanje ima iznos i kut nagiba prema cijevi:

$$a = \sqrt{a_{pN}^2 + \left(a_{pT} + a_c\right)^2} = 338.718\ \text{ms}^{-2}$$

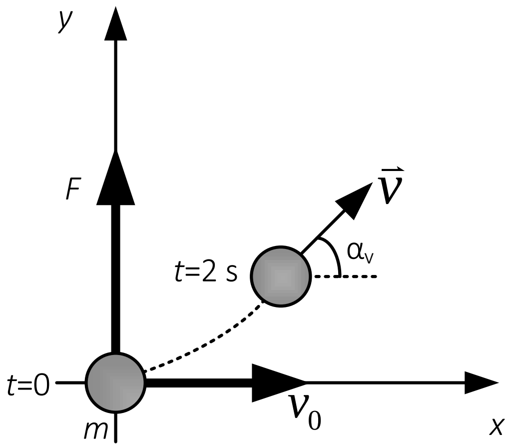
$$\alpha_a = tan^{-1}\left(\frac{a_{pT} + a_c}{a_{pN}}\right) = tan^{-1}\left(\frac{178.024 + 157.080}{49.348}\right) = 81.623°$$

2. DINAMIKA

2.1. Čestica mase m = 0.5 kg giba se u početnom trenutku brzinom v_0 = 10 ms^{-1}. U tom trenutku na česticu počinje djelovati sila okomita na pravac početne brzine. Sila se mijenja po linearnom zakonu tako da joj iznos nakon 2 s poraste s nule na 4 N i ne mijenja se po smjeru. Odrediti brzinu čestice na kraju druge sekunde gibanja.

Rješenje:

Prema zadatku zakon promjene sile $F = 2t$. Pod pretpostavkom da je $\vec{v}_0$ u pravcu osi x, a F se poklapa s osi y bit će:



$$0 = ma_x$$

$$2t = 0.5a_y$$

$$2t = 0.5a_y \quad \Rightarrow \quad 0.5dv_y = 2tdt$$

$$\int 0.5dv_y = 2\int tdt$$

$$0.5v_y = 2\frac{t^2}{2} + C \quad \rightarrow \quad v_y = 2t^2 + C$$

U pravcu osi x nema promjene brzine. Brzina je stalnog iznosa v = 10 m/s.

Za t = 0 s $\longrightarrow$ $v_y = 0 \quad \Longrightarrow C = 0$

Za t = 2 s $\longrightarrow$ $v_y = 8 \text{ m/s}$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{10^2 + 8^2} = 12.81 \text{ m/s}$$

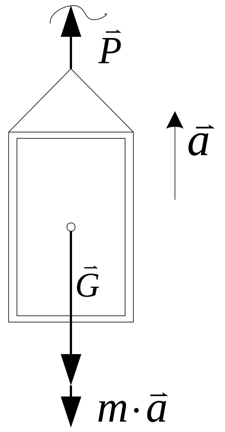
$$\alpha_v = tan^{-1}\left(\frac{v_y}{v_x}\right) = tan^{-1}\left(\frac{8}{10}\right) = 38.66\ °$$

2.2. Lift težine *G* podiže se ili spušta ubrzanjem *a*. Naći silu u konopcu. Kad će ta sila biti nula?

Rješenje:

$m \cdot a = P - G$

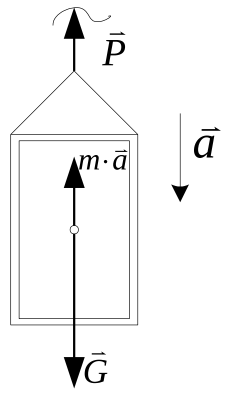
$P = G + m \cdot a = G\left(1 + \frac{a}{g}\right)$



$m \cdot a = G - P$

$P = G\left(1 - \frac{a}{g}\right)$

Sila je jednaka nuli ako je $a = g$



2.3. Materijalna točka giba se pravocrtno pod djelovanjem konstantne sile F = 10 N i prevali put s = 20 m za t = 2 s. Kolika je težina materijalne točke ?

Rješenje:

Iz II. Newtonovog zakona $\sum \vec{\boldsymbol{F}} = \boldsymbol{m} \cdot \vec{\boldsymbol{a}}$

$$F - m \cdot a = 0$$

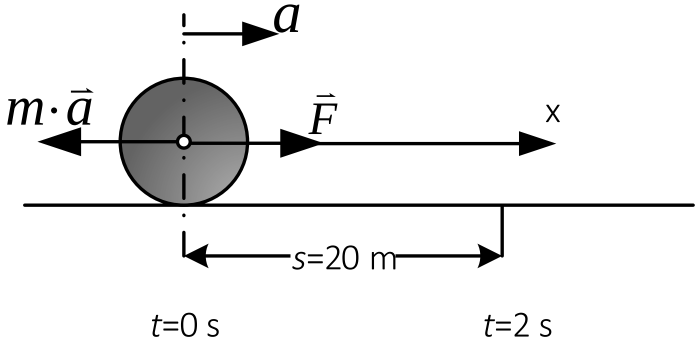
$$F = m \cdot a = \frac{G}{g} \cdot a$$

$$G = F \cdot \frac{g}{a}$$

$$s = s_0 + v_0 t + \frac{a}{2} t^2$$

$$a = \frac{2s}{t^2}$$

$$G = F \cdot \frac{g \cdot t^2}{2s} = 10 \cdot \frac{9.81 \cdot 4^2}{2 \cdot 20} = 9.81 \text{ N}$$

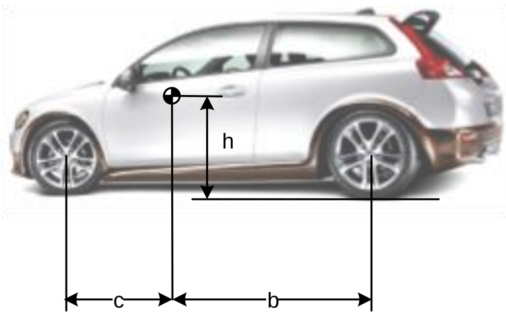


2.4. Auto se kreće po horizontalnoj cesti. Odrediti maksimalno ubrzanje ako automobil dobiva pogon od zadnjih kotača. Zadano: $\boldsymbol{\mu = 0.4}$

$$\boldsymbol{b = 1\ \mathrm{m}}$$

$$\boldsymbol{c = 2\ \mathrm{m}}$$

$$\boldsymbol{h = 0.8\ \mathrm{m}}$$



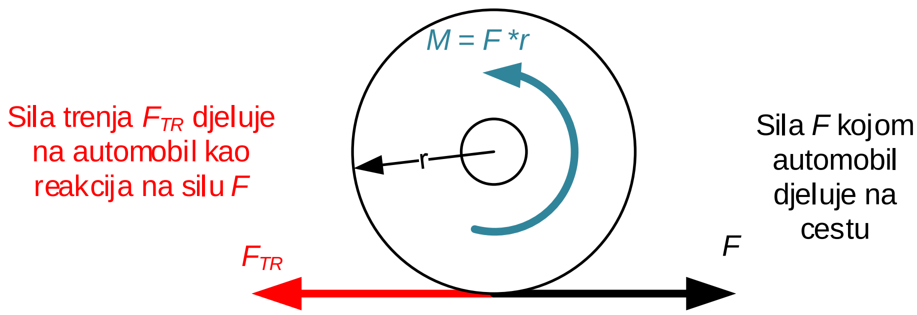
Rješenje:

Jedina vanjska sila koja djeluje u smjeru gibanja automobila je TRENJE !!!

Trenje uvijek djeluje u suprotnom smjeru od gibanja tijela! U slučaju automobila kotači su tijelo koje se giba relativno u odnosu na automobil. S obzirom da je sila na obodu u tom slučaju usmjerena prema natrag, trenje djeluje prema naprijed!

Postoji više vrsta trenja, ovdje će biti riječ o statičkom i kinematičkom. Trenje koje svladavamo kad pokušavamo pokrenuti tijelo u mirovanju naziva se statičko trenje, a jednom kad pokrenemo tijelo potrebna je puno manja sila za održavanje gibanja, odnosno dalje svladavamo kinematičko trenje koje je uvijek manje od statičkog trenja.

Automobil koristi upravo kinematičko trenje da bi postigao ubrzanje *a*, koje ovisi isključivo o koeficijentu trenja. Što je veći koeficijent kinematičkog trenja, automobil ima veće ubrzanje!!! Koeficijent trenja ovisi isključivo o materijalu dviju površina koje se dodiruju.

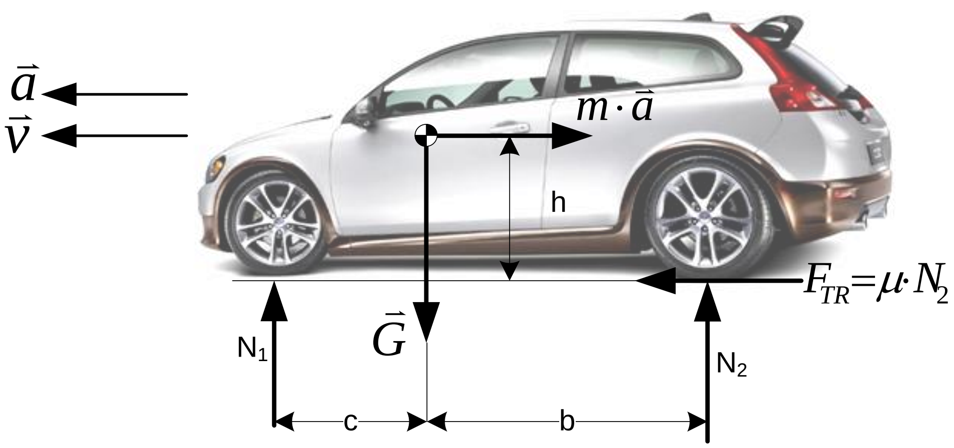


Ako automobil ima pogon na zadnje kotače, okretni moment prenosi se na osovinu kotača. Moment se može zamijeniti silom na obodu kotača koja množi polumjer kotača.

Dok automobil miruje, između kotača i tla ne djeluje sila trenja. Kad pokušavamo gurati automobil počinje djelovati statička sila trenja koja je proporcionalna normalnoj sili (pritisku između automobila i tla) i koeficijentu statičkog trenja koji ovisi o materijalu i stanju podloge i gume kotača. Ako povećavamo silu kojom guramo automobil, u jednom trenutku svladat ćemo silu trenja i automobil će se početi gibati. Nakon toga više nije potrebna tolika sila da bi se automobil nastavio gibati jer više ne svladavamo statičko, već kinematičko trenje koje je uvijek manje od statičkog. Koeficijent trenja kinematičkog gibanja je manji od statičkog, a određuje se eksperimentalno. Kad jednom pokrenemo automobil, reakcija na obodnu silu kotača je kinematičko trenje koje je usmjereno prema naprijed i jedina je sila u pravcu gibanja automobila, ako zanemarimo silu otpora zraka.

Prema prvom Newtonovom zakonu svaka akcija ima reakciju. Dakle, ako je okretanje kotača akcija, sila trenja je reakcija. Što je moment okretanja kotača veći, veća je i sila kinematičkog trenja, pa se automobil ubrzava.

Slika 1. Bugatti Veyron postiže brzinu od 100 km/h za 2.5 s (ubrzanje mu je 11.1 m/s^2)



$$\sum F_x = 0 \qquad F_{tr} = m \cdot a$$

$$\sum M_{N_1} = 0 \qquad -G \cdot c - mah + N_2(c + b) = 0$$

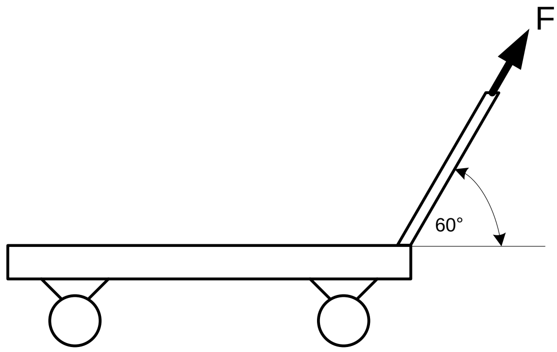
$$a = \frac{F_{tr}}{m} = \frac{\mu N_2}{m} \quad \Rightarrow \quad N_2 = \frac{a \cdot m}{\mu}$$

$$-G \cdot c - m \cdot a \cdot h + \frac{a \cdot m}{\mu}(c + b) = 0$$

$$a \cdot \left[\frac{m}{\mu}(c + b) - m \cdot h\right] = G \cdot c$$

$$a = \frac{G \cdot c}{\frac{m}{\mu}(c + b) - m \cdot h} = \frac{g \cdot c \cdot \mu}{b + c - \mu h} = \frac{9.81 \cdot 2 \cdot 0.4}{1 + 2 - 0.4 \cdot 0.8} = 2.93 \ \mathrm{m/s^2}$$

2.5. Koliki rad obavimo vukući kolica silom od 45 N za ručku nagnutu prema horizontali za $\boldsymbol{\alpha = 60}$°. Put je 300 m.



Rješenje:

$$W = F \cdot cos\, 60° \cdot s = 45 \cdot \frac{1}{2} \cdot 300 = 6\,750 \ \mathrm{Nm}$$

2.6. Tijelo se okreće pod djelovanjem konstantne sile *F* = 60 N okomito na polumjer vrtnje. Hvatišna točka sile je na udaljenosti *R* = 50 cm od osi okretanja i giba se po zakonu $\boldsymbol{s = 6t + 2t^2} \ [\mathbf{cm}]$. Izračunati rad sile za vrijeme prve 3 sekunde okretanja.

Rješenje:

$$s = 6 \cdot 3 + 2 \cdot 3^2 = 36 \ \mathrm{cm}$$

$$\varphi = \frac{s}{R} = \frac{36}{50} = 0.72$$

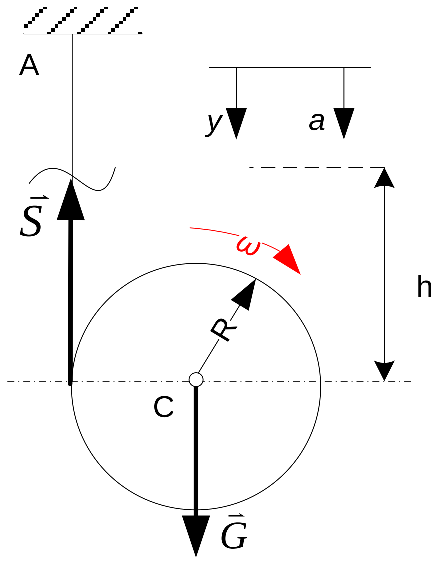
$$W = M \cdot \varphi = F \cdot R \cdot \varphi = 60 \cdot 0.5 \cdot 0.72 = 21.6 \ \mathrm{Nm}$$

2.7. S kružnog valjka polumjera *R* i težine *G*, odmotava se uže čiji je jedan kraj pričvršćen u točki A. Odrediti:

a) brzinu koju će valjak imati pri spuštanju pod utjecajem svoje težine s visine *h*, ako je u početnom trenutku mirovao,

b) ubrzanje valjka pri spuštanju,

c) silu u užetu.



Rješenje:

Zakon o promjeni kinetičke energije:

$U \quad t = 0 \qquad v_c = 0\,; \quad \varepsilon = 0$

$$\frac{1}{2}mv_c^2 + \frac{1}{2}I_c\omega^2 = mgh \qquad\qquad I_c = \frac{1}{2}mR^2 \quad \text{...moment inercije valjka}$$

$$v_c = R \cdot \omega$$

$$\frac{1}{2}mv_c^2 + \frac{1}{2}\cdot\frac{1}{2}mR^2\frac{{v_c}^2}{R^2} = mgh$$

$$\frac{3}{4}mv_c^2 = mgh$$

$$v_c = \sqrt{\frac{4gh}{3}}$$

b)

$$\sum y = 0 \qquad m \cdot a_c = G - S \quad (1)$$

$$\sum M_c = 0 \qquad I_c \cdot \dot{\omega} = S \cdot R \quad (2)$$

$$\varepsilon = \dot{\omega} = \frac{a_c}{R}$$

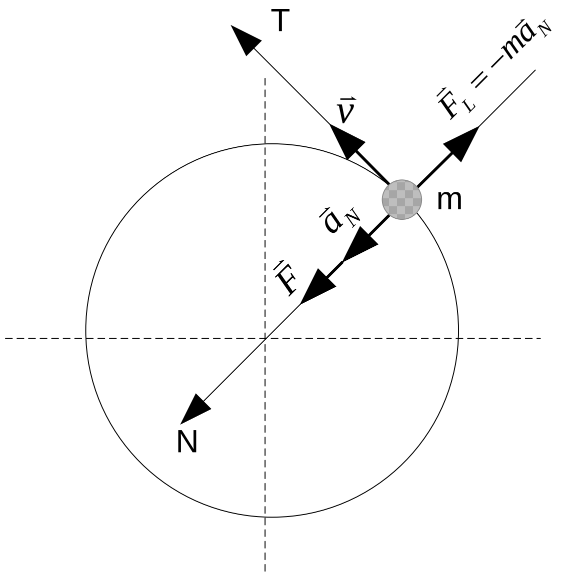
$$\frac{1}{2} m R^2 \cdot \frac{a_c}{R} = S \cdot R \quad (2) \quad \Longrightarrow \quad S = \frac{1}{2} m a_c$$

$$a_c = g - \frac{S}{m} = g - \frac{1}{m}\frac{1}{2} m a_c \quad (2) \text{ u } (1)$$

$$\frac{3}{2} a_c = g \quad \Longrightarrow \quad a_c = \frac{2}{3} g$$

$$S = \frac{1}{2} m \frac{2}{3} g = \frac{1}{3} m g = \frac{G}{3}$$

2.8. Pri gibanju čestice po kružnoj putanji konstantnim iznosom brzine *v,* na česticu djeluje samo normalno (centripetalno) ubrzanje $\boldsymbol{a_N = v^2/R}$ kojemu nasuprot djeluje centrifugalna sila $\boldsymbol{F_L = m a_N = m v^2/R}$.



Rješenje:

Prema D'Alambertovom principu bit će $F - mv^2/R = 0$ pa je $F = mv^2/R$.

Za takvo je gibanje potrebna sila F stalno usmjerena prema središtu zakrivljenosti putanje (centripetalna sila). Ovdje je centripetalna sila ujedno i središnja jer je središte zakrivljenosti nepomično središte gibanja.

2.10. **Automobil sa blokiranim kočnicama klizi po mokroj cesti (koeficijent trenja μ = 0.2). U trenutku kada počinje kočenje brzina automobila iznosila je 60 km/h. Koliki je put zaustavljanja automobila?**

Rješenje:

Zakon kinetičke energije:

$$\frac{mv_2^2}{2} - \frac{mv_1^2}{2} = \int_1^2 \vec{F} \cdot d\vec{r} = W$$

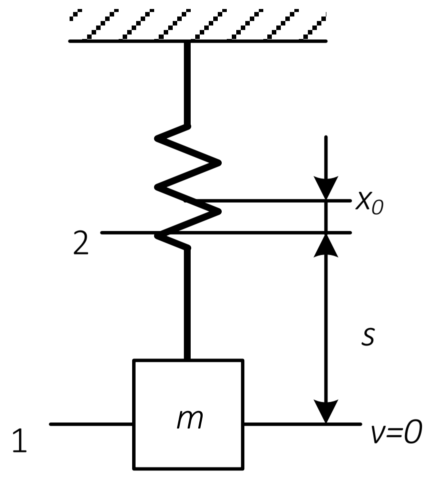
$$v_2 = 0, \ \ a \ \ v_1 = 60 \ \text{km/h} = 16.7 \ \text{m/s}$$

Sila trenja je konstantna $F_T = \mu m g$

$$-\frac{mv_1^2}{2} = -F_T s = \mu m g s$$

$$s = \frac{v_1^2}{2\mu g} = \frac{16.7^2}{2 \cdot 0.2 \cdot 9.81} = 70.8 \ \text{m}$$

2.11. **Na opruzi konstantne krutosti *c* obješen je uteg mase *m*. Pod djelovanjem utega opruga se produži za x_0 (ravnotežni položaj). Ako se uteg pomakne prema dolje za dužinu *s* i iz tog položaja ispusti, odrediti brzinu kojom će uteg projuriti kroz ravnotežni položaj?**



Rješenje:

Potencijalna energija u krajnjem donjem položaju (položaj 1) ukupna je energija tog položaja i iznosi:

$$E_1 = \frac{c(x_0 + s)^2}{2}$$

U ravnotežnom položaju (položaj 2) ukupna energija E_2 sastoji se od kinetičke energije čestice, potencijalne energije položaja čestice na visini s u odnosu na položaj 1 i potencijalne energije opruge pa je:

$$E_2 = \frac{mv^2}{2} + mgs + \frac{c{x_0}^2}{2}$$

Sve su sile na česticu konzervativne (teža, sila opruge), tako da je u svakom položaju zbroj mehaničkih energija isti, tj. $E_2 = E_1$, što daje:

$$\frac{c(x_0+s)^2}{2} = \frac{mv^2}{2} + mgs + \frac{c{x_0}^2}{2}$$

$$\frac{c(x_0+s)^2}{2} = \frac{mv^2}{2} + mgs + \frac{c{x_0}^2}{2} \quad /\cdot 2$$

$$c(x_0+s)^2 = mv^2 + 2mgs + c{x_0}^2$$

$$c({x_0}^2 + 2sx_0 + s^2) = mv^2 + 2mgs + c{x_0}^2$$

$$c(2sx_0 + s^2) = mv^2 + 2mgs$$

$$2sx_0c + cs^2 = mv^2 + 2mgs$$

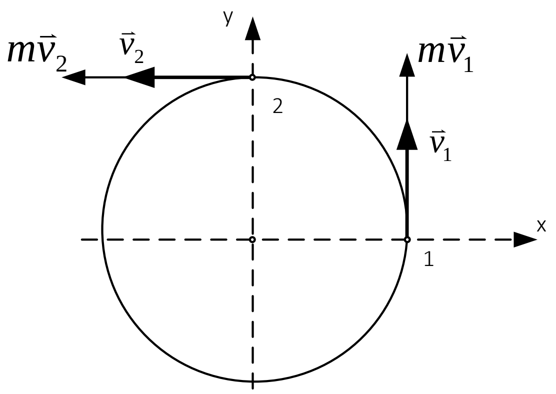
U ravnotežnom položaju je $mg = cx_0$

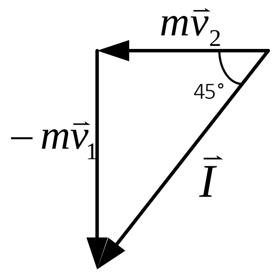
$$2mgs + cs^2 = mv^2 + 2mgs$$

$$cs^2 = mv^2 \quad \rightarrow \quad v = s\sqrt{\frac{c}{m}}$$

2.12. Pri gibanju čestice mase *m* konstantnim iznosom brzine po kružnoj putanji polumjera *R* impuls sile je razlika količine gibanja u dva trenutka. Za položaje 1 i 2 bit će:

$$\boldsymbol{m\vec{v}_2 - m\vec{v}_1 = \vec{I}}$$





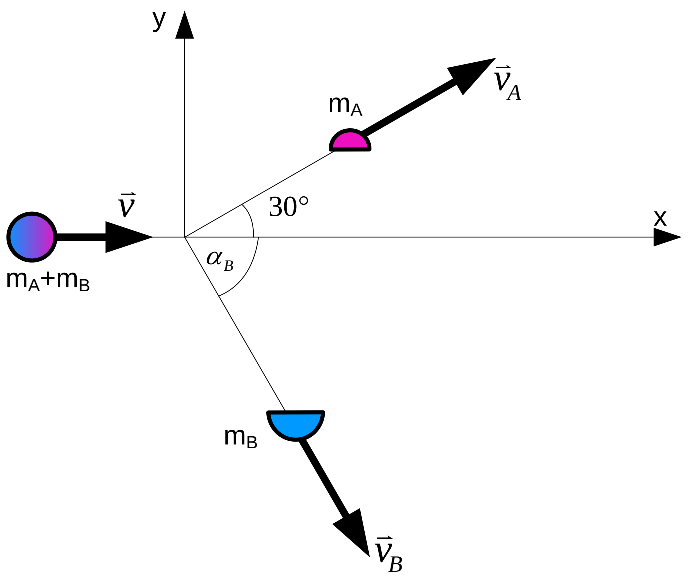
Iz vektorske slike proizlazi da je iznos impulsa $I = mv\sqrt{2}$, a smjer mu je određen kutom od 225° prema x osi. Ili ako promatramo jednadžbe u x i y smjeru:

$$-mv_{2x} = I_x \quad \Rightarrow I_x = -mv$$

$$mv_{2y} = I_y \qquad \Rightarrow I_y = mv$$

$$I = \sqrt{{I_x}^2 + {I_y}^2} = \sqrt{(-mv)^2 + (mv)^2} = \sqrt{2(mv)^2} = mv\sqrt{2}$$

2.13. Pod djelovanjem unutrašnjih sila došlo je do odvajanja čestice m_A = 2 kg od mase m_B= 8 kg, prema slici. Zajednička je brzina prije odvajanja iznosila v = 100 m/s. Čestica m_A odvojila se tako da joj je nakon odvajanja brzina bila v_A = 433 m/s pod kutom od 30° prema pravcu prvobitnog gibanja. Odrediti gibanje druge čestice neposredno nakon odvajanja i rad unutrašnjih sila potreban za odvajanje čestica.



Rješenje:

Kako su pri odvajanju djelovale samo unutrašnje sile, količina gibanja sustava prije i nakon odvajanja mora ostati nepromijenjena:

$$(m_A + m_B)\vec{v} = m_A\vec{v}_A + m_B\vec{v}_B$$

Ta jednadžba daje dvije skalarne jednadžbe (projekcije na os x i y):

$$(m_A + m_B)v = m_A v_A \cos 30° + m_B v_{Bx} \qquad (1)$$

$$0 = m_A v_A \sin 30° - m_B v_{By} \qquad (2)$$

$$iz \quad (2) \qquad v_{By} = \frac{m_A}{m_B} v_A \sin 30° = \frac{2}{8} \cdot 433 \cdot \sin 30° = 54.13 \text{ m/s}$$

$$v_{Bx} = \frac{(m_A + m_B)}{m_B} v - \frac{m_A}{m_B} v_A \cos 30° = \frac{(2+8)}{8} \cdot 100 - \frac{2}{8} \cdot 433 \cdot \cos 30° = 31.39 \text{ m/s}$$

$$v_B = \sqrt{{v_{Bx}}^2 + {v_{By}}^2} = \sqrt{31.39^{\,2} + 54.13^2} = 62.5\ \mathrm{m/s}$$

$$\alpha_B = tan^{-1}\frac{v_{By}}{v_{Bx}} = tan^{-1}\frac{54.13}{31.39} = 60\ °$$

Prema zakonu kinetičke energije, rad unutrašnjih sila jednak je razlici kinetičkih energija sustava nakon i prije odvajanja:

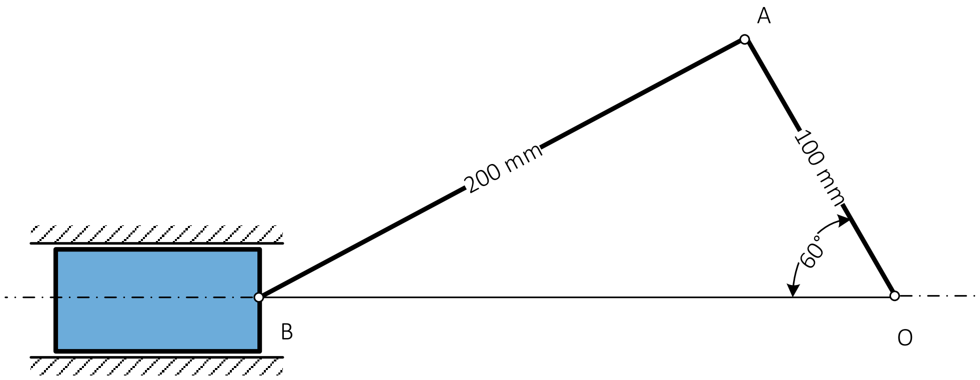
$$E_{k2} - E_{k1} = W = W_{Fu}$$

$$\frac{m_A {v_A}^2}{2} + \frac{m_B {v_B}^2}{2} - \frac{(m_A + m_B)v^2}{2} = W_{Fu}$$

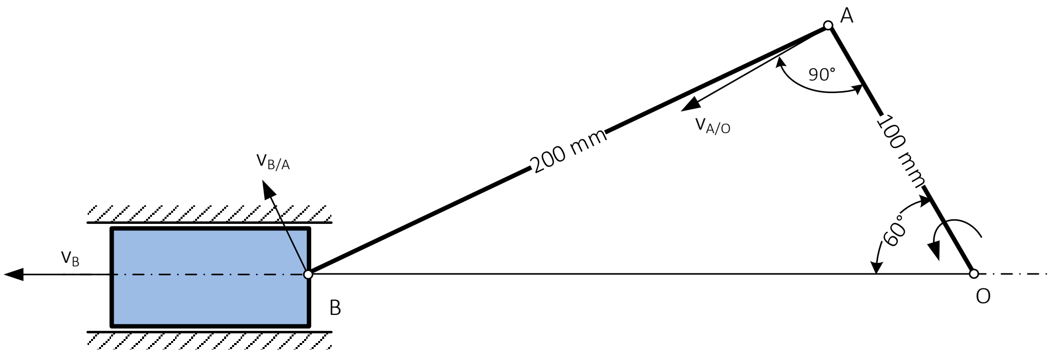
$$W_{Fu} = \frac{2 \cdot 433^2}{2} + \frac{8 \cdot 62.5^2}{2} - \frac{(2+8) \cdot 100^2}{2} = 153.2\ \mathrm{kJ}$$

3. TEORIJA MEHANIZAMA

3.1. Poluga OA rotira u smjeru suprotnom kazaljci na satu sa 3000 o/min. Potrebno je odrediti brzinu klipa i kutnu brzinu poluge AB oko točke A?



Rješenje:

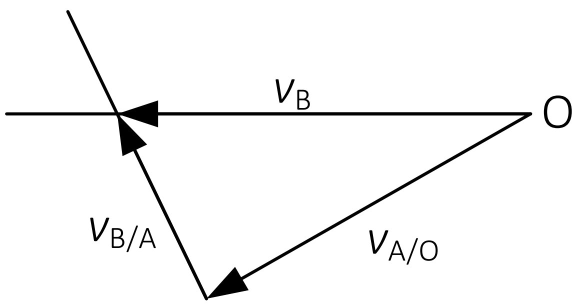


$$\omega = \frac{2\pi N}{60} = 314.2 \ \text{rad/s}$$

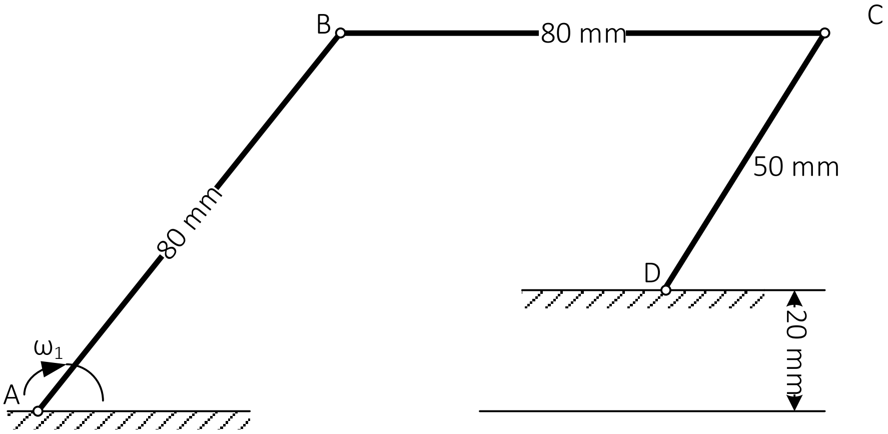
$$v_{A/O} = \overline{OA} \cdot \omega = 0.1 \cdot 314.2 = 31.4 \ \text{m/s}$$

$v_B = 34.7 \ \text{m/s}$ ...očitano iz dijagrama brzina

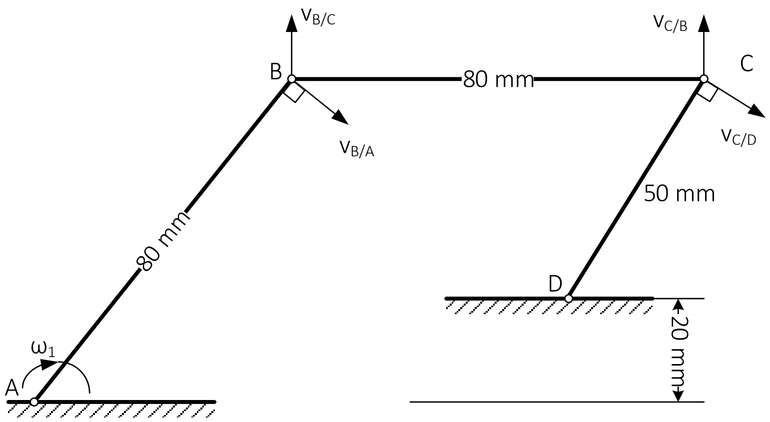
$$\omega_{AB} = \frac{v_{B/A}}{\overline{AB}} = \frac{17.4}{0.2} = 87 \ \text{rad/s}$$



3.2. Odredi kutnu brzinu štapa DC i smjer rotacije, ako je kutna brzina $\boldsymbol{\omega_1 = 500\ \mathrm{o/min}}$. Štap BC je horizontalan.



Rješenje:

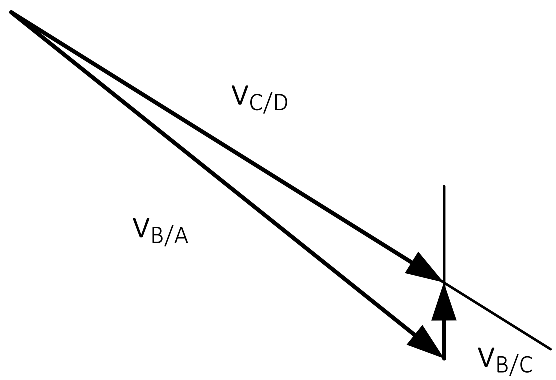


$$\omega_1 = \frac{2\pi N}{60} = \frac{2\pi \cdot 500}{60} = 52.36\ \text{rad/s}$$

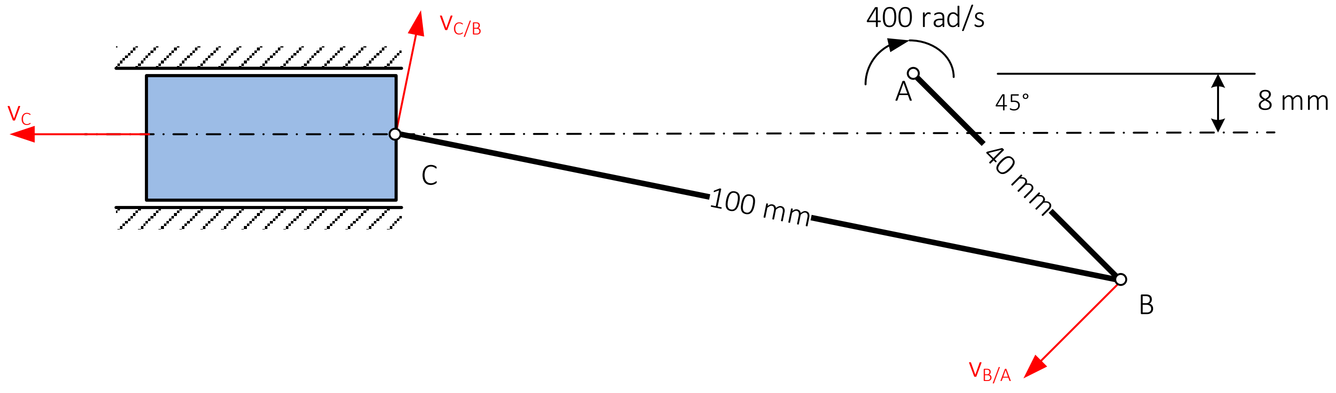
$$v_{B/A} = \overline{AB} \cdot \omega_1 = 0.08 \cdot 52.36 = 4.2\ \text{m/s}$$

$v_{C/D} = 3.8\ \text{m/s}$ ...očitano iz dijagrama brzina

$$\omega_{CD} = \frac{v_{C/D}}{\overline{CD}} = \frac{3.8}{0.05} = 76\ \text{rad/s}$$



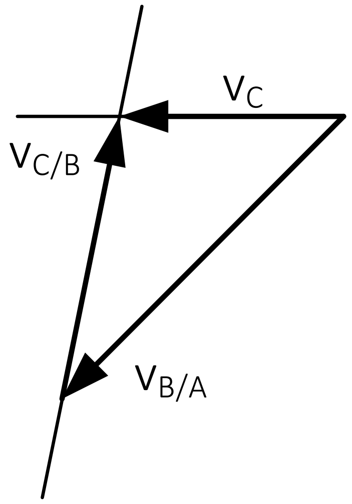
3.3. Klip prikazanog mehanizma ima masu 0.8 kg. Odredi njegovu akceleraciju i silu inercije potrebnu za poziciju prikazanu na slici.

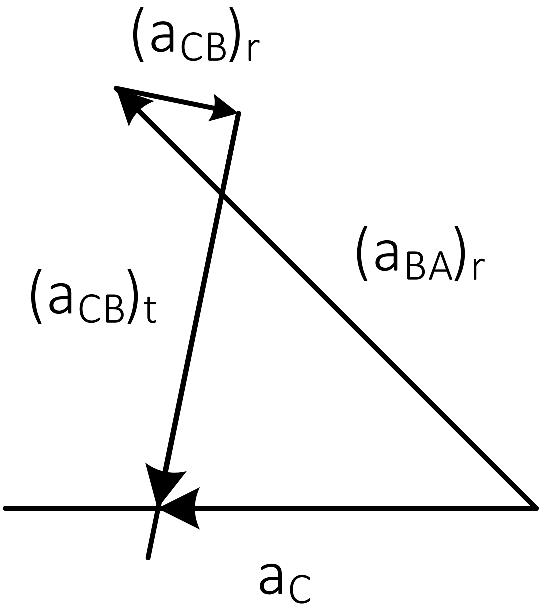


Rješenje:

$\omega = 400 \ \mathrm{rad/s}$

$v_{B/A} = \overline{AB} \cdot \omega = 0.04 \cdot 400 = 16 \ \mathrm{m/s}$





Očitano:

$v_C = 9 \ \mathrm{m/s}$

$v_{C/B} = 11.6 \ \mathrm{m/s}$

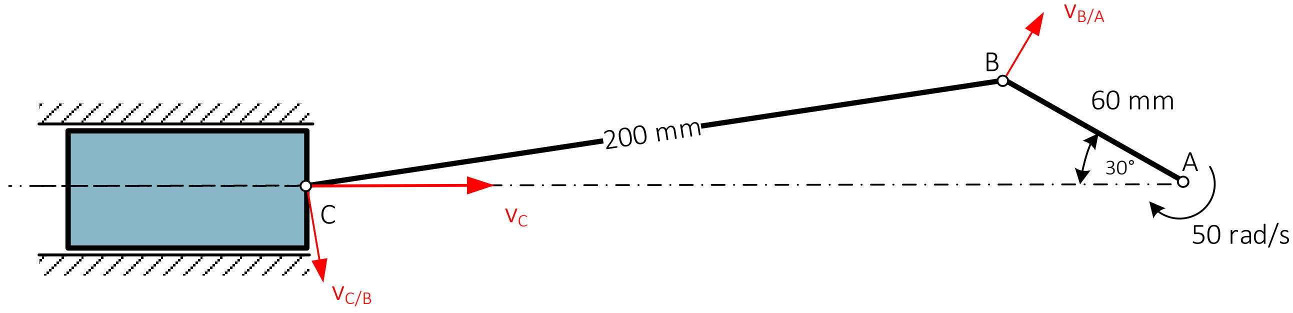
$$(a_{BA})_r = \overline{AB} \cdot \omega^2 = \frac{{v_{B/A}}^2}{\overline{AB}} = \frac{16^2}{0.04} = 6400 \ \mathrm{m/s^2}$$

$$(a_{CB})_r = \frac{{v_{C/B}}^2}{\overline{BC}} = \frac{11.6^2}{0.1} = 1345.6 \ \mathrm{m/s^2}$$

$a_C = 4080 \ \mathrm{m/s^2}$ ... očitano iz dijagrama ubrzanja

$F = m \cdot a_C = 0.8 \cdot 4080 = 3264 \ \mathrm{N}$

3.4. Odredi ubrzanje klipa na slici!

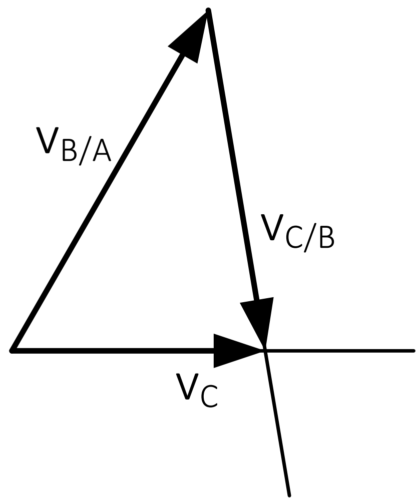


Rješenje:

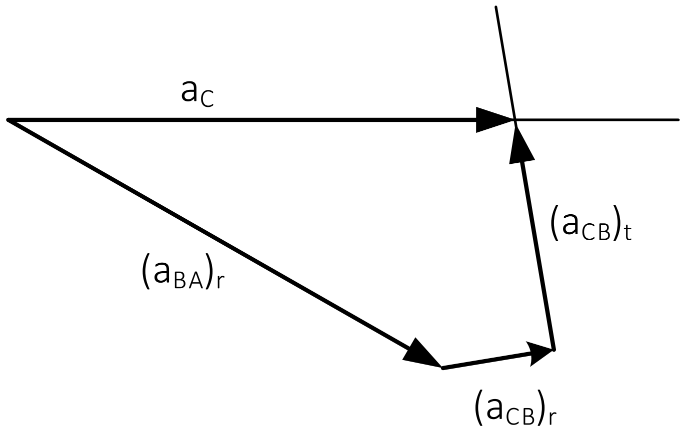
$\omega = 50 \ \text{rad/s}$

$v_{B/A} = \overline{BA} \cdot \omega = 0.06 \cdot 50 = 3 \ \text{m/s}$

Dijagram brzina



Dijagram ubrzanja



$v_C = 1.9 \ \text{m/s}$

$$(a_{BA})_r = \frac{{v_{B/A}}^2}{\overline{AB}} = \frac{3^2}{0.06} = 150 \ \text{m/s}^2$$

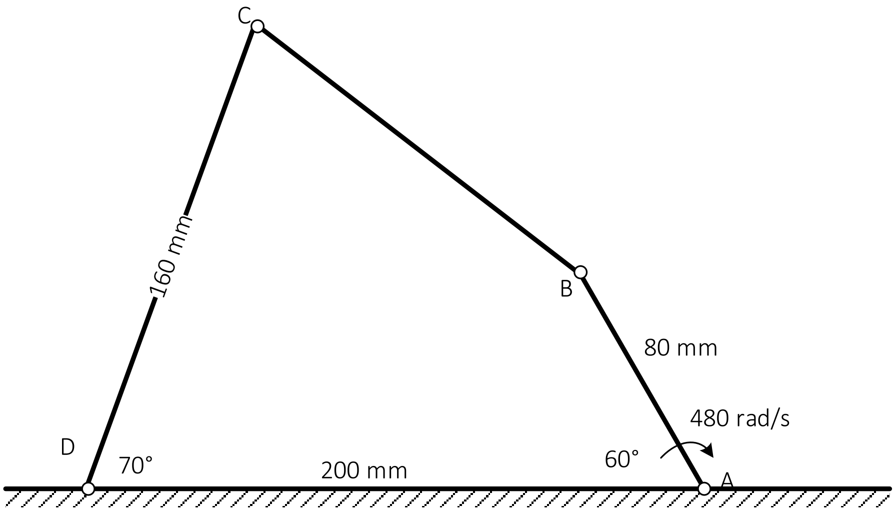
$$(a_{CB})_r = \frac{{v_{C/B}}^2}{\overline{CB}} = \frac{2.6^2}{0.2} = 33.8 \ \text{m/s}^2$$

$a_C = 151.5 \ \text{m/s}^2$ … očitano iz dijagrama ubrzanja

$(a_{CB})_t = 69.3 \ \text{m/s}^2$

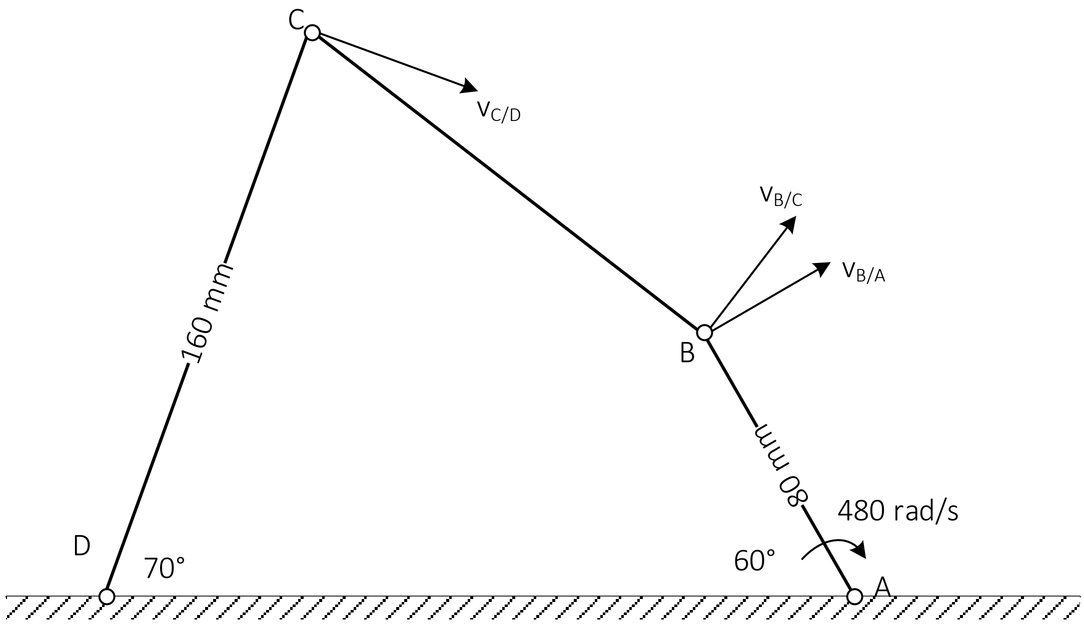
$$\varepsilon_{BC} = \frac{(a_{CB})_t}{\overline{BC}} = \frac{69.3}{0.2} = 346.5 \ \text{rad/s}^2$$

3.5. Odrediti kutno ubrzanje štapa CD za prikazani mehanizam.



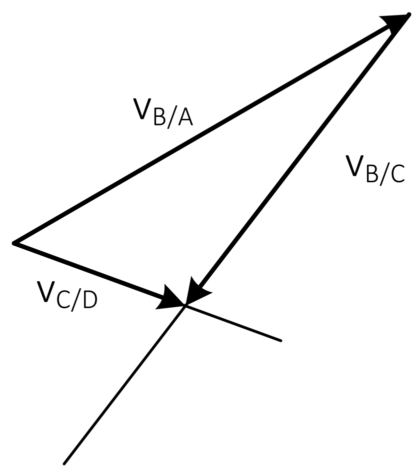
Rješenje:

$\overline{CB} = 136\ \text{mm}$ … izmjereno sa slike



$\omega_1 = 480\ \text{rad/s}$

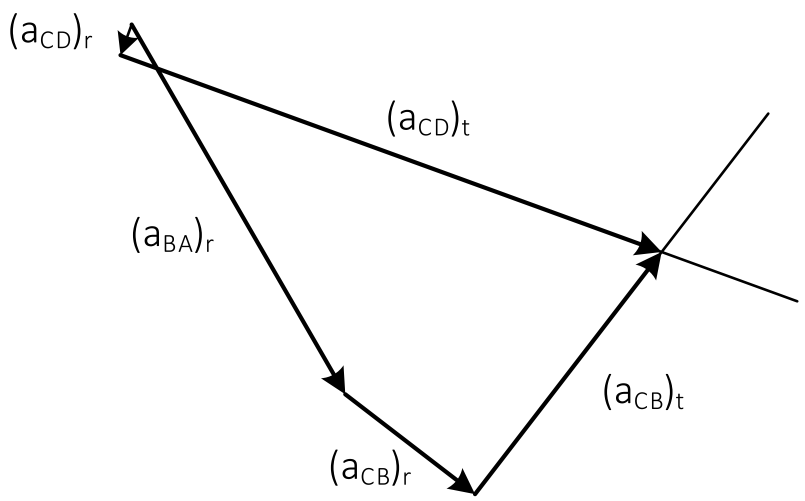
$v_{B/A} = \overline{AB} \cdot \omega_1 = 0.08 \cdot 480 = 38.4\ \text{m/s}$



$$(a_{BA})_r = \frac{{v_{B/A}}^2}{\overline{AB}} = \frac{38.4^2}{0.08} = 18\,432\ \mathrm{m/s^2}$$

$$(a_{CD})_r = \frac{{v_{C/D}}^2}{\overline{CD}} = \frac{15^2}{0.16} = 1406\ \ \mathrm{m/s^2}$$

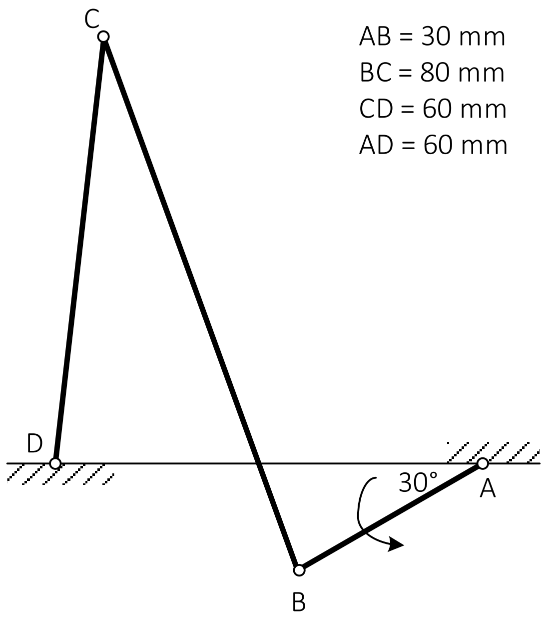
$$(a_{CB})_r = \frac{{v_{C/B}}^2}{\overline{CB}} = \frac{31^2}{0.136} = 7066\ \ \mathrm{m/s^2}$$



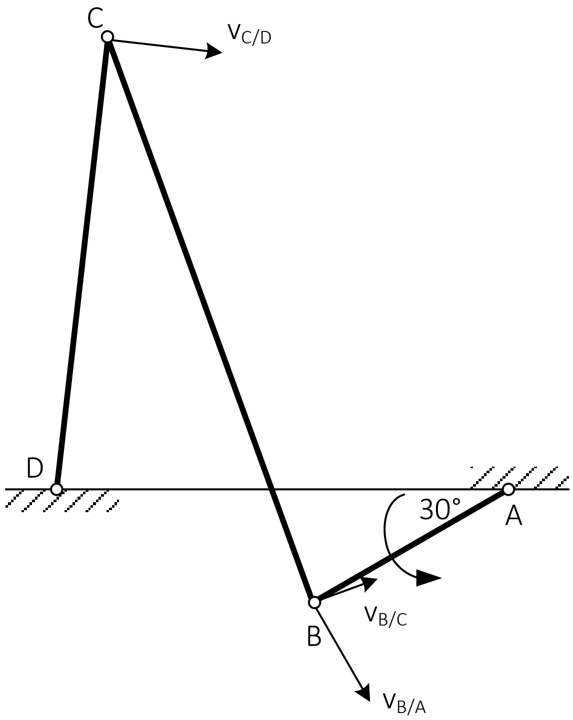
$$(a_{CD})_t = 24\,000\ \ \mathrm{m/s^2}$$

$$\varepsilon_{CD} = \frac{(a_{CD})_t}{\overline{CD}} = \frac{24000}{0.16} = 150\,000\ \ \mathrm{rad/s^2}$$ ...suprotno kazaljci na satu

3.6. Slika prikazuje četvero zglobni mehanizam. Štap AB rotira konstantnom brzinom od 5 rad/s suprotno kazaljci na satu. Potrebno je odrediti kutno ubrzanje štapa DC za prikazani položaj.



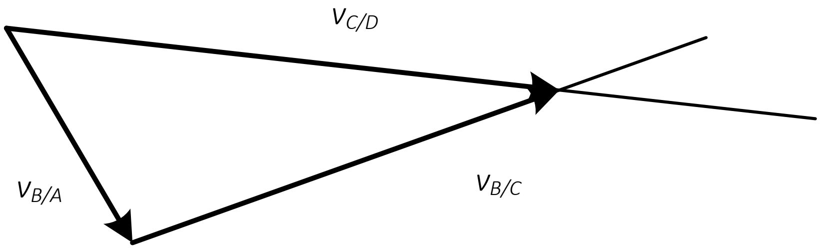
Rješenje:



$\omega = 5 \ \text{rad/s}$

$v_{B/A} = \overline{AB} \cdot \omega = 0.03 \cdot 5 = 0.15 \ \text{m/s}$

Dijagram brzina

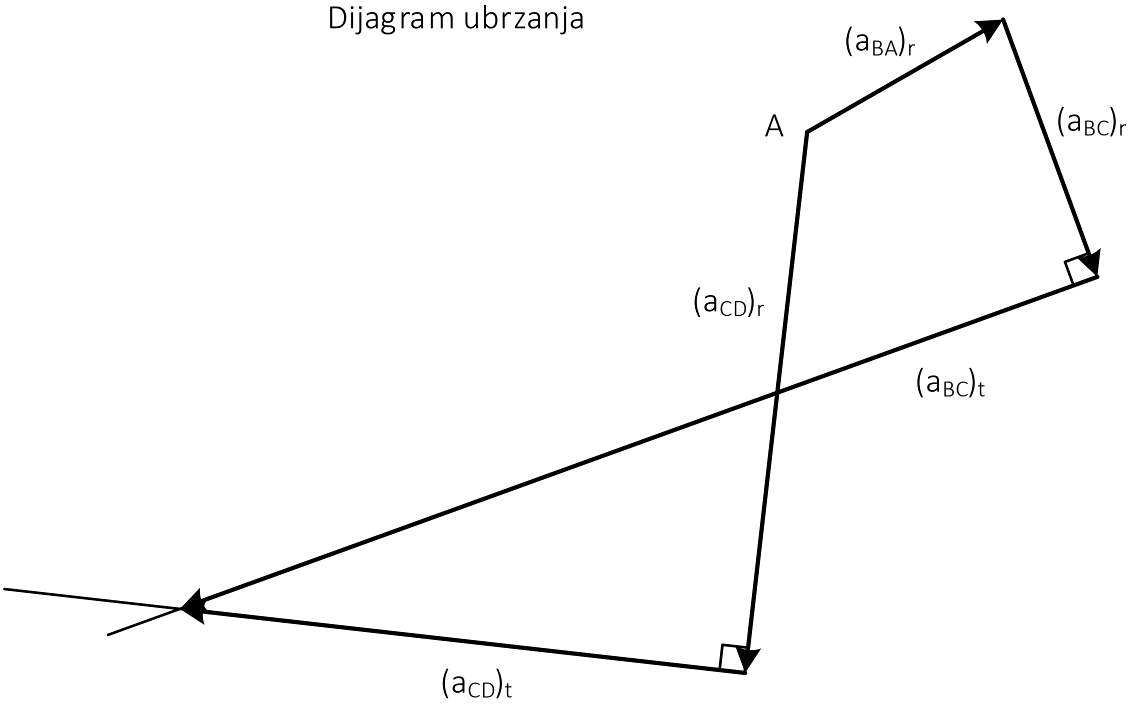


$$(a_{BA})_r = \frac{{v_{B/A}}^2}{\overline{AB}} = \frac{0.15^2}{0.03} = 0.75 \text{ m/s}^2$$

$$(a_{CD})_r = \frac{{v_{C/D}}^2}{\overline{CD}} = \frac{0.33^2}{0.06} = 1.815 \text{ m/s}^2$$

$$(a_{CB})_r = \frac{{v_{C/B}}^2}{\overline{CB}} = \frac{0.27^2}{0.08} = 0.91 \text{ m/s}^2$$

Dijagram ubrzanja

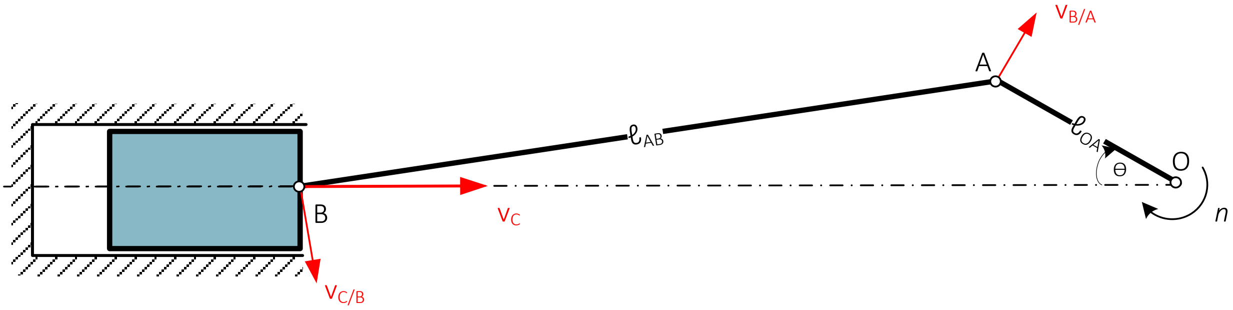


$(a_{CD})_t = 1.8 \text{ m/s}^2$

$\varepsilon_{CD} = \frac{(a_{CD})_t}{\overline{CD}} = \frac{1.8}{0.06} = 30 \text{ rad/s}^2$ … suprotno kazaljci na satu

3.7. Za klipni mehanizam prema slici u određenom trenutku poznati su sljedeći podaci:

duljina radilice OA	$\ell_{OA} = 30$ cm
duljina vratila AB	$\ell_{AB} = 150$ cm
brzina vrtnje radilice OA	n=150 o/min
kut zakreta radilice	$\Theta = 135°$
tlak u cilindru u promatranom trenutku	$p = 1155000$ Pa
promjer klipa	$\phi D = 12$ cm
masa klipa	$m = 1{,}8$ kg



Odrediti:

a) nacrtati mehanizam u zadanom položaju u mjerilu,

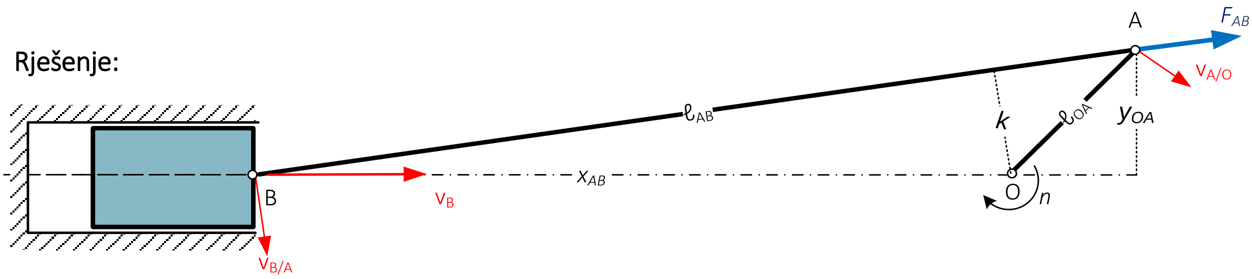
b) nacrtati dijagram brzina,

c) nacrtati dijagram ubrzanja,

d) odrediti brzinu i ubrzanje klipa (točka B),

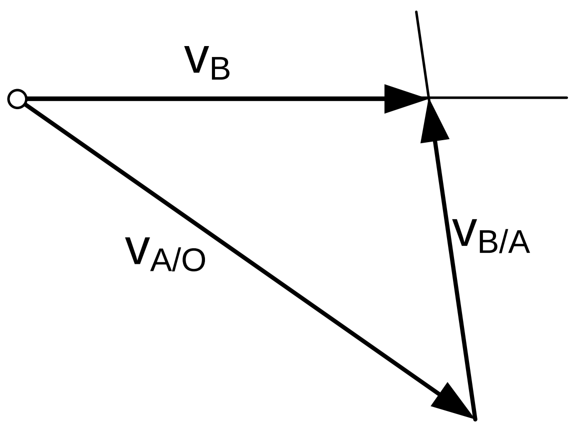
e) odrediti silu inercije i rezultantnu silu na klip.

Rješenje:



$$\omega = \frac{150 \cdot 2\pi}{60} = 15.71 \text{ rad/s}$$

$$v_{B/A} = \overline{OA} \cdot \omega = 0.30 \cdot 15.71 = 4.71 \text{ m/s}$$

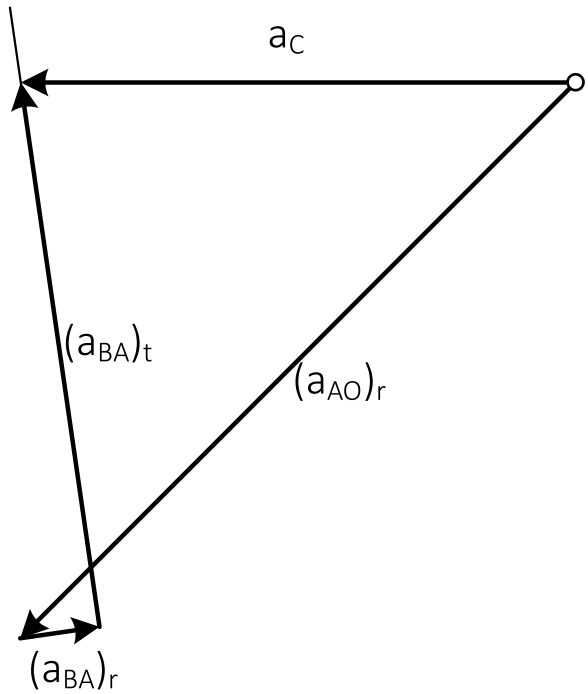


$$(a_{OA})_r = \frac{{v_{A/O}}^2}{\overline{OA}} = \frac{4.71^2}{0.30} = 74.02\,\mathrm{m/s^2}$$

$$(a_{BA})_r = \frac{{v_{B/A}}^2}{\overline{AB}} = \frac{15^2}{1.50} = 7.55\ \mathrm{m/s^2}$$

$$(a_{BA})_t = 51.8\ \mathrm{m/s^2}$$

Dijagram ubrzanja



$$a_B = -52.2\ \mathrm{m/s^2}$$

$$F_p = p \cdot \frac{D^2\pi}{4} = 1155000 \cdot \frac{0.12^2\pi}{4} = 13062.7\ \mathrm{N}$$

$$F_{in} = m \cdot a_B = 1.8 \cdot -52.2 = -93.9\ \mathrm{N}$$

$$F_B = F_p - F_{in} = 13062.7 + 93.9 = 13156.7\ \mathrm{N}$$

$$F_{AB} = F_B \cdot \cos\gamma = 13024.5\ \mathrm{N}$$

$$\cos\gamma = \frac{x_{AB}}{\overline{AB}} = \frac{\sqrt{\overline{AB}^2 - y_{AO}^2}}{\overline{AB}} = \frac{\sqrt{1.5^2 - 0.212^2}}{1.5} = 0.9899$$

$$y_{AO} = AO \cdot \sin(\theta) = 0.30 \cdot \sin(135°) = 0.212\ \mathrm{m}$$

$$k = \overline{OA} \cdot \sin(\gamma + \theta) = 0.18\ \mathrm{m}$$

$$M_{AO} = F_{AB} \cdot k = 13024.5 \cdot 0.18 = 2344.4\ \mathrm{Nm}$$

4. JEDNOLIKI HORIZONTALNI LET ZRAKOPLOVA

4.1. Odredite potreban potisak i snagu na razini mora u uvjetima standardne atmosfere ISA/SL za zrakoplove Cessna Skylane i Cessna Citation 3 u ravnotežnom horizontalnom letu, te brzine za najmanju potrebnu snagu i silu potiska?

Tablica 4-1. Podaci za zrakoplov Cessna Skylane i Cessna Citation 3

AVION	*b* [m]	*A* [m^2]	*m* [kg]	C_{D0}	*e*	$\boldsymbol{AR = \frac{b^2}{A}}$
Cessna Skylane 	10.9	16.2	1338.1	0.025	0.8	7.3
Cessna Citation 3 	16.2	29.5	8987.9	0.02	0.81	8.9

$$F_{TR} = F_D = \underbrace{\frac{1}{2}\rho V^2}_{q_\infty} C_D A = \frac{1}{2}\rho V^2 A(C_{D0} + C_{Di})$$

$$F_{TR} = \frac{1}{2}\rho V^2 A\left(C_{D0} + \frac{{C_L}^2}{\pi e AR}\right)$$

$$F_L = F_G \qquad \Rightarrow \qquad \frac{1}{2}\rho V^2 C_L A = F_G$$

$$F_{TR} = \frac{1}{2} C_{D0}\,\rho A \cdot V^2 + \frac{2{F_G}^2}{\pi e AR \rho A}\frac{1}{V^2}$$

SKYLANE

Potrebna sila potiska F_{TR}:

$$F_{TR} = \frac{1}{2} \cdot 0.025 \cdot 1.225 \cdot 16.2 \cdot V^2 + \frac{2(1338.1 \cdot 9.81)^2}{\pi \cdot 0.8 \cdot 7.3 \cdot 1.225 \cdot 16.2} \frac{1}{V^2}$$

$$F_{TR} = 0.248\, V^2 + 0.947 \cdot 10^6 \frac{1}{V^2}$$

Potrebna snaga P_R:

$$P_R = F_{TR} \cdot V$$

$$P_R = 0.248\, V^3 + 0.947 \cdot 10^6 \frac{1}{V}$$

Najmanja potrebna sila potiska $F_{TR\,min}$:

$$F_{TR_{min}} = \frac{dF_{TR}}{dV} = 0 \quad \Longrightarrow \qquad 0.496\, V - 1.894 \cdot 10^6 \frac{1}{V^3} = 0 / \cdot V^3$$

$$0.496\, V^4 = 1.894 \cdot 10^6$$

$$V^4 = \frac{1.894 \cdot 10^6}{0.496}$$

$$\boxed{V_{F_{TR_{min}}} = 44.2 \quad \mathrm{m/s}} \quad \Longrightarrow \quad \boxed{F_{TR_{min}} = 969.2 \ \mathrm{N}}$$

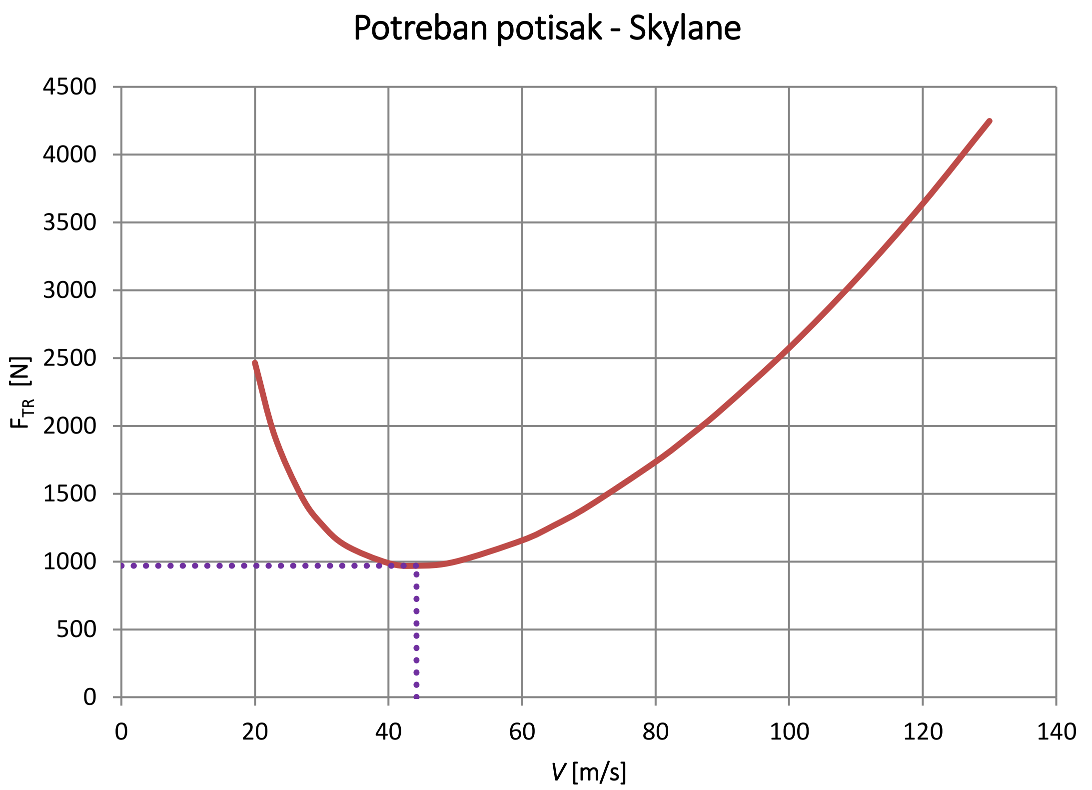
Najmanja potrebna snaga $P_{R\,min}$:

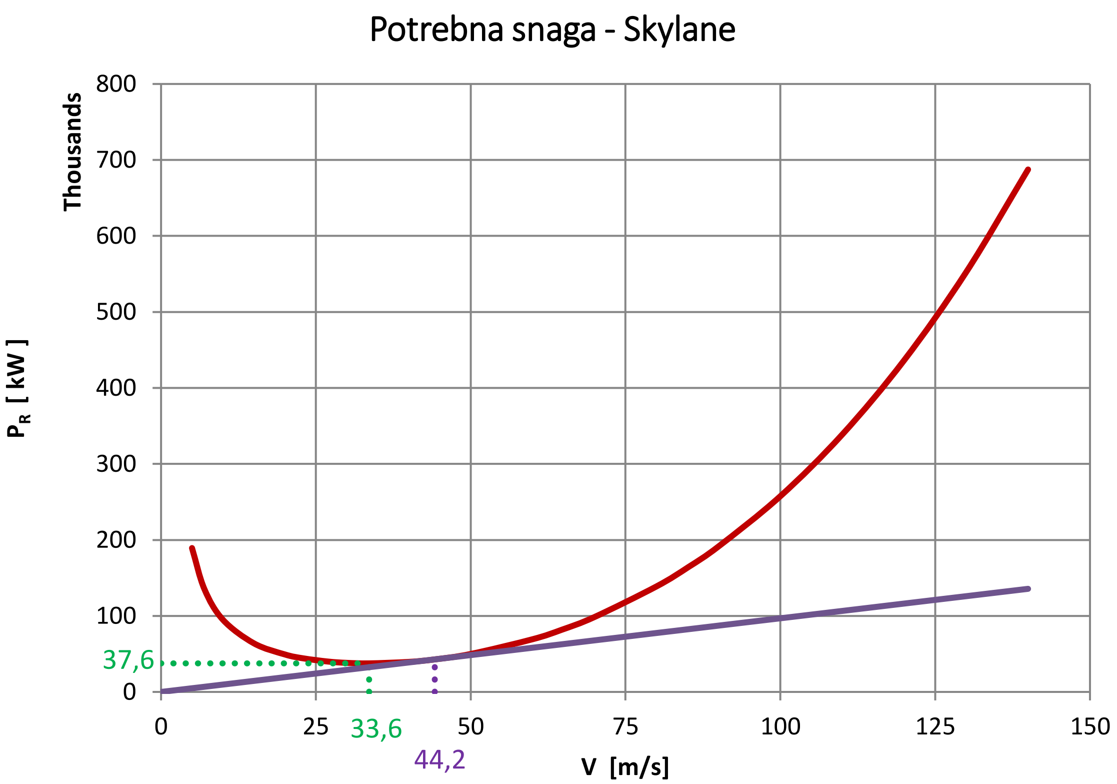
$$P_{R_{min}} = \frac{dP_R}{dV} = 0$$

$$0.744\, V^2 - 0.947 \cdot 10^6 \frac{1}{V^2} = 0 / \cdot V^2$$

$$V^4 = \frac{0.947 \cdot 10^6}{0.744}$$

$$\boxed{V_{P_{R_{min}}} = 33.6 \quad \mathrm{m/s}} \quad \Longrightarrow \quad \boxed{P_{R_{min}} = 0.248 \cdot 33.6^3 + 0.947 \cdot 10^6 \cdot \frac{1}{33.6} = 37.6 \ \mathrm{kW}}$$


Potreban potisak - Skylane
4500
4000
3500
3000
2500
2000
1500
1000
500
0
F_{TR} [N]
0
20
40
60
80
100
120
140
V [m/s]


Potrebna snaga - Skylane
800
700
600
500
400
300
200
100
37,6
0
Thousands
P_R [kW]
0
25
33,6
50
44,2
75
100
125
150
V [m/s]

CITATION

Potrebna sila potiska $\boldsymbol{F_{TR}}$:

$$F_{TR} = \frac{1}{2} \cdot 0.02 \cdot 1.225 \cdot 29.5 \cdot V^2 + \frac{2(8987.9 \cdot 9.81)^2}{\pi \cdot 0.81 \cdot 8.9 \cdot 1.225 \cdot 29.5} \cdot \frac{1}{V^2}$$

$$F_{TR} = 0.361\, V^2 + 19 \cdot 10^6 \frac{1}{V^2}$$

Potrebna snaga P_R:

$$P_R = F_{TR} \cdot V$$

$$P_R = 0.361\, V^3 + 19 \cdot 10^6 \frac{1}{V}$$

Najmanja potrebna sila potiska $F_{TR\,min}$:

$$F_{TR_{min}} = \frac{dF_{TR}}{dV} = 0$$

$$0.722\, V - 38 \cdot 10^6 \frac{1}{V^3} = 0 / \cdot V^3$$

$$0.722\, V^4 = 38 \cdot 10^6$$

$$V^4 = \frac{38 \cdot 10^6}{0.722}$$

$$\boxed{V_{F_{TRmin}} = 85.2 \quad \mathrm{m/s}} \quad \Longrightarrow \quad \boxed{F_{TR_{min}} = 5238\ \mathrm{N}}$$

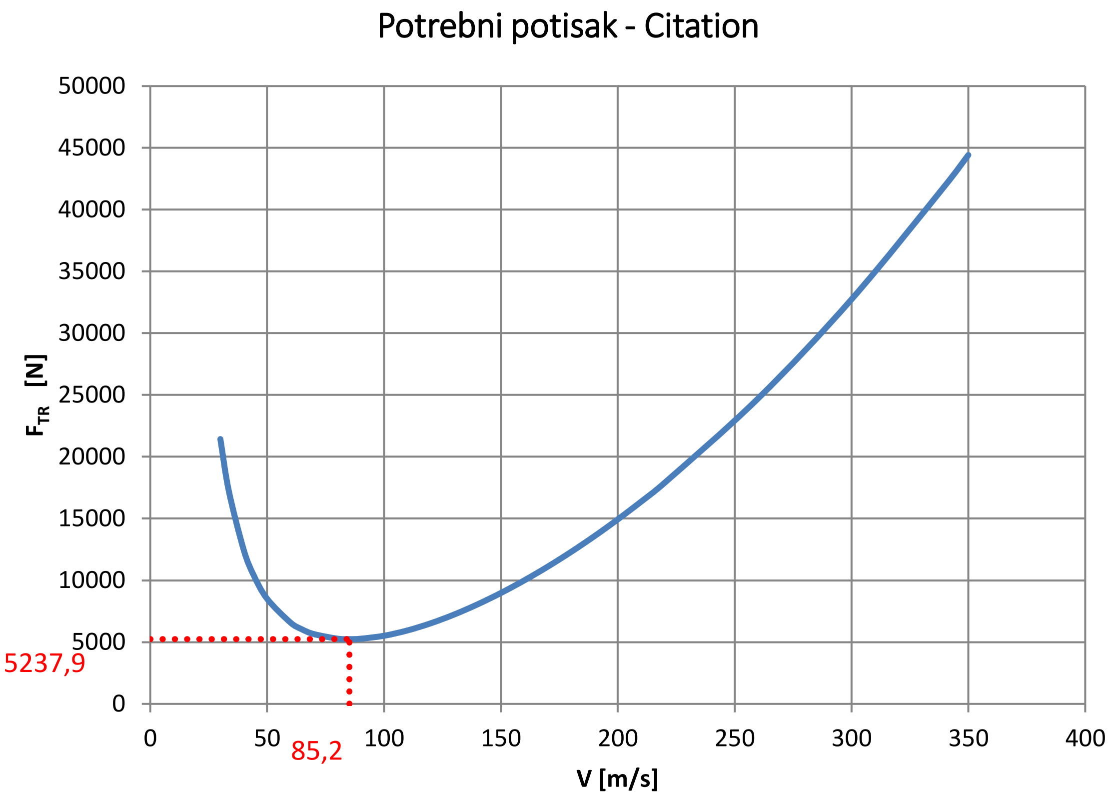
Najmanja potrebna snaga $P_{R\,min}$:

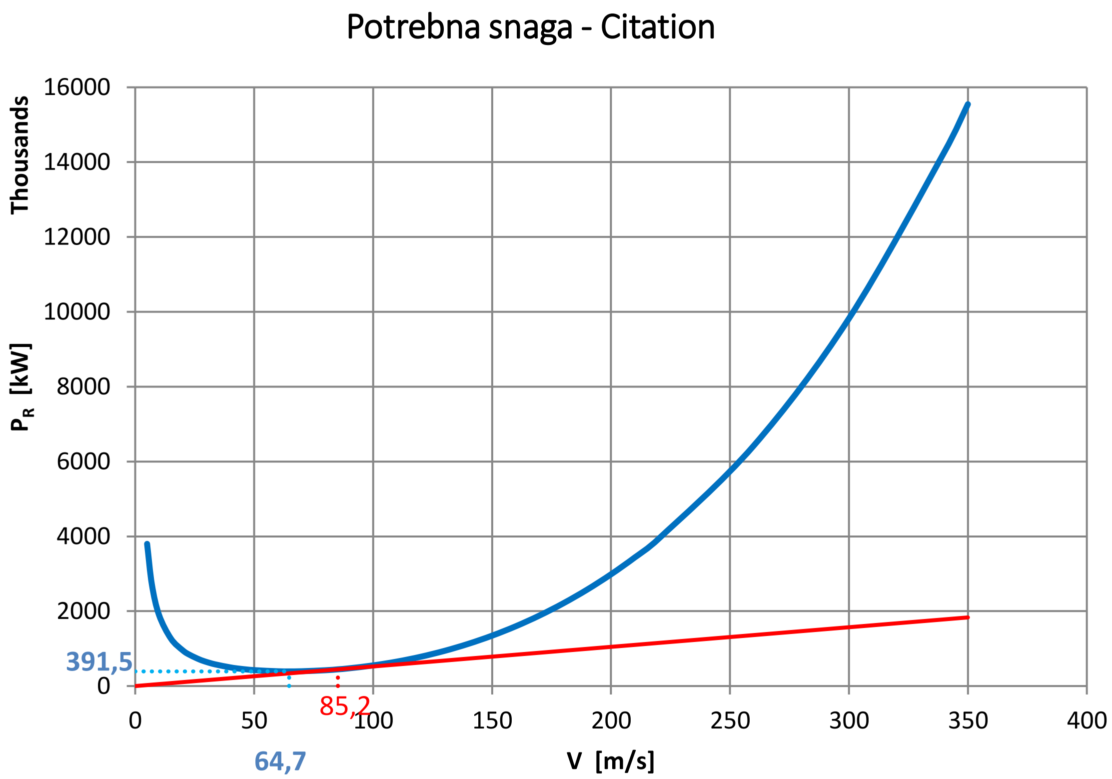
$$P_{R_{min}} = \frac{dP_R}{dV} = 0$$

$$1.083\, V^2 - 19 \cdot 10^6 \frac{1}{V^2} = 0 / \cdot V^2$$

$$V^4 = \frac{19 \cdot 10^6}{1.083}$$

$$\boxed{V_{P_{R_{min}}} = 64.7 \quad \mathrm{m/s}} \quad \Longrightarrow \quad \boxed{P_{R_{min}} = 0.361 \cdot\ 64.7^3 + 19 \cdot 10^6 \frac{1}{64.7} = 391.5\ \ \mathrm{kW}}$$


Potrebni potisak - Citation
50000
45000
40000
35000
30000
25000
20000
15000
10000
5000
0
5237,9
F_TR [N]
0
50
85,2
100
150
200
250
300
350
400
V [m/s]


Potrebna snaga - Citation
Thousands
16000
14000
12000
10000
8000
6000
4000
2000
0
391,5
P_R [kW]
0
50
85,2
100
150
200
250
300
350
400
64,7
V [m/s]

4.2. Odredi brzinu, potrebnu snagu i potisak za uvjet najboljeg doleta pri planiranju za zrakoplove Cessna Skylane i Cessna Citation 3.

$$f_{max} = \left(\frac{C_L}{C_D}\right)_{max} \quad \rightarrow \quad \left(\frac{C_L}{C_{D0} + \frac{{C_L}^2}{\pi eAR}}\right)' = \frac{C_{D0} + \frac{{C_L}^2}{\pi eAR} - C_L \cdot \frac{2C_L}{\pi eAR}}{\left(C_{D0} + \frac{{C_L}^2}{\pi eAR}\right)^2} = 0$$

$$C_{D0} - \frac{{C_L}^2}{\pi eAR} = 0 \quad \Rightarrow \quad C_L = \sqrt{C_{D0}\pi eAR}$$

$$C_D = C_{D0} + \frac{{C_L}^2}{\pi eAR} = 2C_{D0}$$

Skylane

$$C_L = \sqrt{0.025 \cdot \pi \cdot 0.8 \cdot 7.3} = 0.677$$

$$C_D = 2 \cdot 0.025 = 0.05$$

$$F_L = F_G$$

$$\frac{1}{2}\rho V^2 C_L A = mg \quad \rightarrow \quad V = \sqrt{\frac{2mg}{C_L\, \rho\, A}} = \sqrt{\frac{2 \cdot 1338.1 \cdot 9.81}{0.677 \cdot 1.225 \cdot 16.2}} = 44.2 \ \ \mathrm{m/s}$$

$$P_R = F_D \cdot V = C_D \frac{1}{2}\rho V^3 A$$

$$P_R = 0.05 \cdot \frac{1}{2} \cdot 1.225 \cdot 44.2^3 \cdot 16.2 = 42.8 \ \ \mathrm{kW}$$

$$F_{TR} = \frac{P_R}{V} = \frac{42840}{44.2} = 969.2 \quad N$$

$$Provjera: \quad F_{TR} = \frac{C_D}{C_L} F_G = \frac{0.05}{0.677} \cdot 1338.1 \cdot 9.81 = 969.5 \ \ \mathrm{N}$$

Citation

$C_L = \sqrt{0.02 \cdot \pi \cdot 0.81 \cdot 8.9} = 0.673$

$C_D = 2 \cdot 0.02 = 0.04$

$F_L = F_G$

$\frac{1}{2}\ \rho V^2 C_L A = mg \quad \rightarrow \quad V = \sqrt{\frac{2mg}{C_L\, \rho\ A}} = \sqrt{\frac{2 \cdot 8987.9 \cdot 9.81}{0.673 \cdot 1.225 \cdot 29.5}} = 85.2\ \ \mathrm{m/s}$

$P_R = F_D \cdot V = C_D \frac{1}{2}\ \rho V^3 A$

$P_R = 0.04 \cdot \frac{1}{2} \cdot 1.225 \cdot 85.2^3 \cdot 29.5 = 447\ \ \mathrm{kW}$

$F_{TR} = \frac{P_R}{V} = \frac{447000}{85.2} = 5246.5\ \ \mathrm{N}$

Provjera: $F_{TR} = \frac{C_D}{C_L} F_G = \frac{0.04}{0.673} \cdot 8987.9 \cdot 9.81 = 5240.5\ \ \mathrm{N}$

4.3. Izračunaj potrebnu snagu za Cessnu Skylane i Citation 3 na visinama 0, 2000 i 4000 m.

$$P_R = \frac{1}{2} C_{D0}\, \rho A \cdot V^3 + \frac{2{F_G}^2}{\pi e A R \rho A} \frac{1}{V}$$

Skylane

$$P_R = \frac{1}{2} 0.025 \cdot 16.2 \cdot \rho V^3 + \frac{2(1338.1 \cdot 9.81)^2}{\pi \cdot 0.8 \cdot 7.3 \cdot 16.2} \frac{1}{\rho V}$$

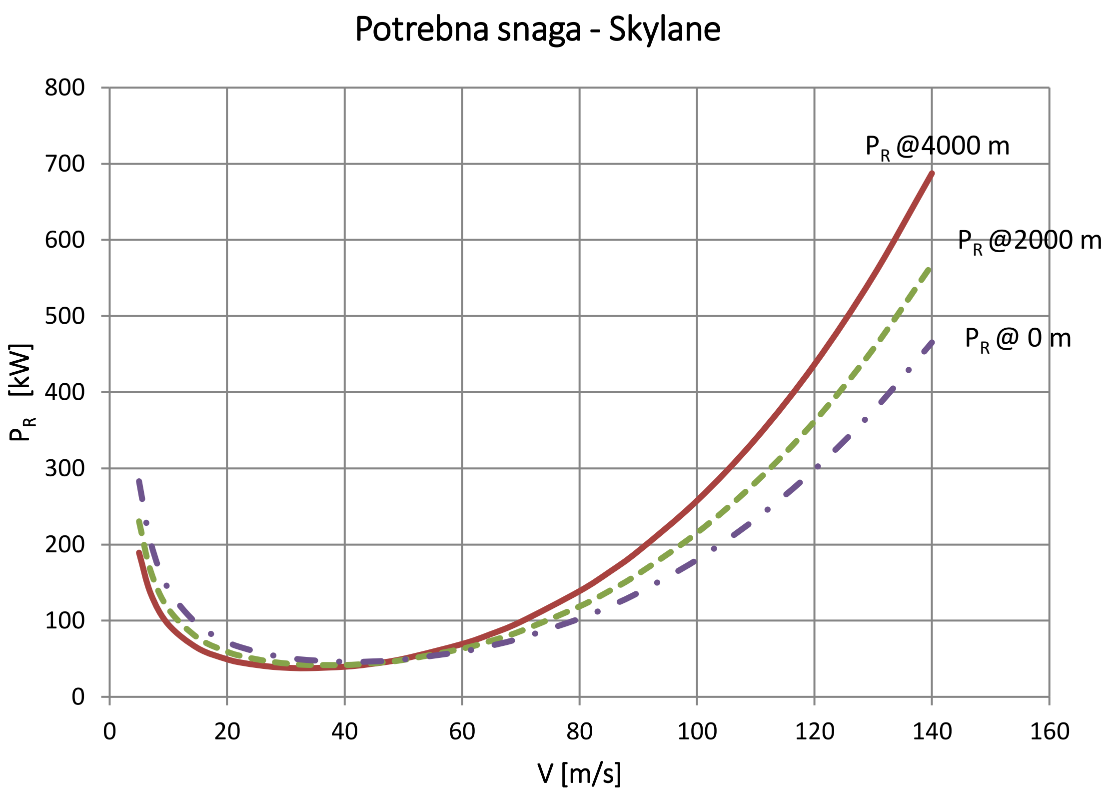
$$P_R = 0.2025 \cdot \rho V^3 + 1.159 \cdot 10^6 \frac{1}{\rho V}$$

$\boldsymbol{H}(\mathbf{m})$	$\boldsymbol{\rho}(\mathbf{kg/m^3})$
0	1.225
2000	1.00656
4000	0.81934

$$P_{R_0} = 0.248 \cdot V^3 + 0.946 \cdot 10^6 \frac{1}{V}$$

$$P_{R_{2000}} = 0.204 \cdot V^3 + 1.151 \cdot 10^6 \frac{1}{V}$$

$$P_{R_{4000}} = 0.166 \cdot V^3 + 1.415 \cdot 10^6 \frac{1}{V}$$



***Citation* 3**

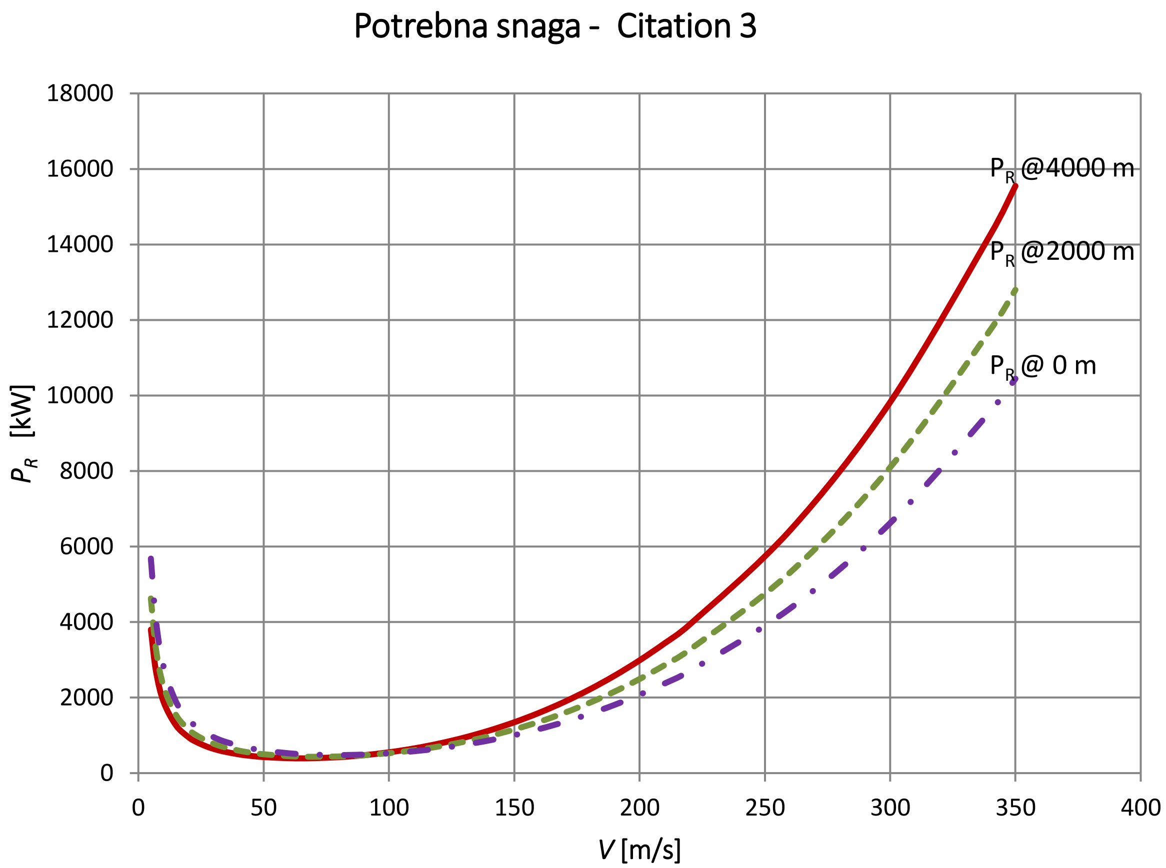
$$P_R = \frac{1}{2} 0.02 \cdot 29.5 \cdot \rho V^3 + \frac{2(8987.9 \cdot 9.81)^2}{\pi \cdot 0.81 \cdot 8.9 \cdot 29.5} \frac{1}{\rho V}$$

$$P_R = 0.295 \cdot \rho V^3 + 23.272 \cdot 10^6 \frac{1}{\rho V}$$

$$P_{R_0} = 0.361 \cdot V^3 + 18.998 \cdot 10^6 \frac{1}{V}$$

$$P_{R_{2000}} = 0.297 \cdot V^3 + 23.120 \cdot 10^6 \frac{1}{V}$$

$$P_{R_{4000}} = 0.242 \cdot V^3 + 28.403 \cdot 10^6 \frac{1}{V}$$



S većom visinom leta raste potrebna snaga za svladavanje parazitnog otpora iznad brzine $V_{(CL/CD)max}$ raste.

4.4. Za zrakoplov mase 5000 kg, površine krila 36 m^2, eksperimentima u aerodinamičkom tunelu određena je ovisnost aerodinamičkih karakteristika:

$\boldsymbol{C_D = 0.025 + 0.045{C_L}^2}$.

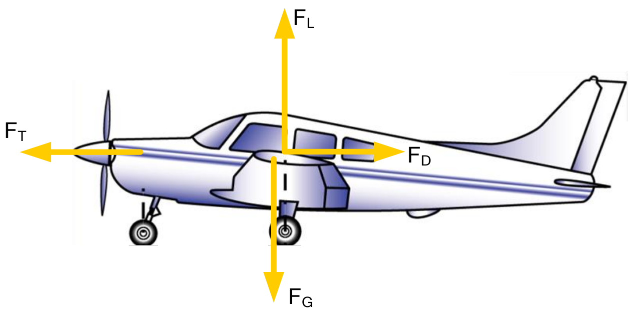
Odredi:

a) funkcionalnu ovisnost potrebne snage o brzini $\boldsymbol{P_R = f(V)}$,

b) potrebnu snagu za ustaljeni horizontalni let brzinom 360 km/h,

c) maksimalnu finesu zrakoplova te

d) nacrtaj Pénaud-ov dijagram snage i označi karakteristične brzine i snage.



a)

$$F_D = F_T$$

$$\frac{1}{2}\rho V^2 C_L A = mg \quad \rightarrow \quad C_L = \frac{2mg}{\rho V^2 A}$$

$$P_R = F_T \cdot V = F_D \cdot V = C_D \frac{1}{2}\rho V^3 A$$

$$P_R = \frac{1}{2}(0.025 + 0.045{C_L}^2)\rho A V^3$$

$$P_R = \frac{1}{2}\left(0.025 + 0.045\frac{4m^2 g^2}{\rho^2 V^4 A^2}\right)\rho A V^3$$

$$P_R = 0.0125\rho A V^3 + 0.09\frac{m^2 g^2}{\rho A}\frac{1}{V}$$

$$P_R = 0.0125 \cdot 1.225 \cdot 36 V^3 + 0.09\frac{5000^2 \cdot 9.81^2}{1.225 \cdot 36}\frac{1}{V}$$

$$\boxed{P_R = 0.55125V^3 + 4.91 \cdot 10^6\frac{1}{V}}$$

II način

$$P_R = F_{TR}V = \frac{C_D}{C_L}F_G V = \frac{0.025 + 0.045{C_L}^2}{C_L}F_G V = \left(\frac{0.025}{C_L} + 0.045C_L\right)F_G V$$

$$F_G = F_L = C_L\frac{1}{2}\rho V^2 A \quad \rightarrow \quad C_L = \frac{2F_G}{\rho V^2 A}$$

$$P_R = \left(\frac{0.025}{\frac{2F_G}{\rho V^2 A}} + 0.045\frac{2F_G}{\rho V^2 A}\right)F_G V$$

$$\boxed{P_R = 0.0125\rho A V^3 + 0.09\frac{{F_G}^2}{\rho A}\frac{1}{V}}$$

b)

$$P_R = 0.55125 \cdot 100^3 + 4.91 \cdot 10^6 \frac{1}{100} = 600350\ \text{W} = 600\ \text{kW}$$

c)

$$f = \frac{C_L}{C_D} = \frac{C_L}{0.025 + 0.045{C_L}^2}$$

$$\frac{df}{dC_L} = \frac{0.025 + 0.045{C_L}^2 - C_L \cdot 0.09C_L}{\left(0.025 + 0.045{C_L}^2\right)^2} = \frac{0.025 - 0.045{C_L}^2}{\left(0.025 + 0.045{C_L}^2\right)^2} = 0$$

$$0.025 - 0.045{C_L}^2 = 0$$

$$0.025 = 0.045{C_L}^2$$

$${C_L}^2 = 0.5556 \quad \rightarrow \quad C_L = 0.745356$$

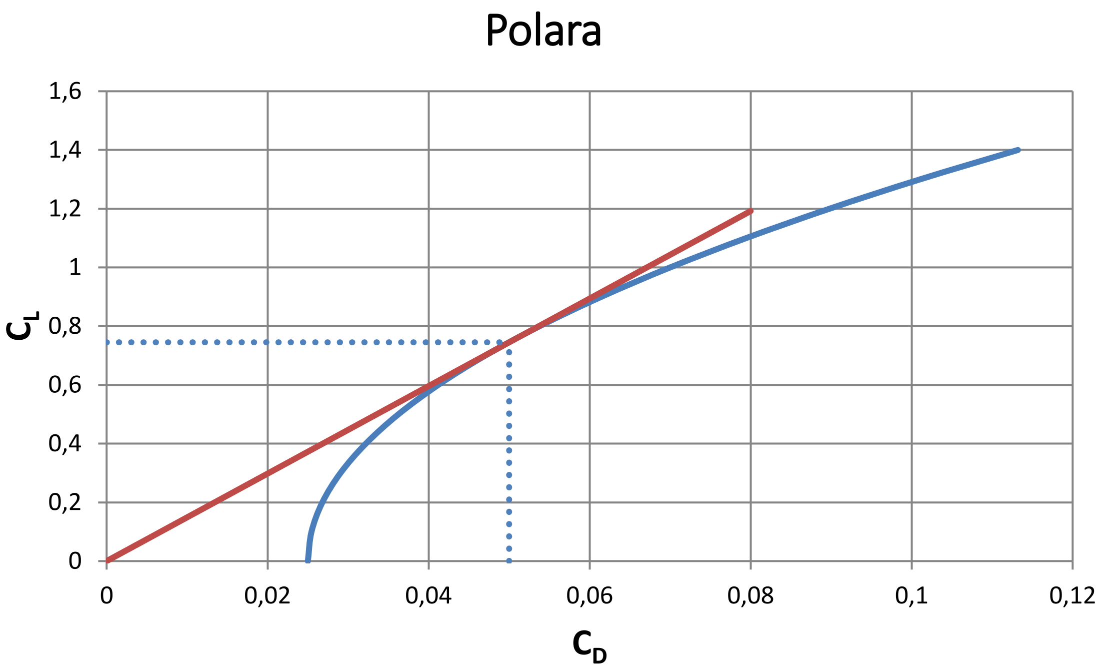
$$C_D = 0.025 + 0.045{C_L}^2 = 0.05$$

$$f = \frac{C_L}{C_D} = \frac{0.745356}{0.05} = 14.9$$

Dijagram → $C_L = 0.745$

$$C_D = 0.05$$

$$f = \frac{C_L}{C_D} = \frac{0.745}{0.05} = 14.9$$



d) Na Pénaud dijagramu za ovaj zrakoplov označi:

d1) $V_{P_{Rmin}}$

$$P_R = 0.55125V^3 + 4.91 \cdot 10^6 \frac{1}{V}$$

$$\frac{dP_R}{dV} = 1.65375V^2 - 4.91 \cdot 10^6 \frac{1}{V^2}$$

$$\frac{dP_R}{dV} = 0 \quad \Longrightarrow \quad 1.65375V^2 - 4.91 \cdot 10^6 \frac{1}{V^2} = 0 / \cdot V^2$$

$$1.65375V^4 = 4.91 \cdot 10^6$$

$$V^4 = 2.969 \cdot 10^6$$

$$\boxed{V_{P_{Rmin}} = 41.5 \quad \mathrm{m/s}}$$

$$P_{R_{min}} = 0.55125 {V_{P_{Rmin}}}^3 + 4.91 \cdot 10^6 \frac{1}{V_{P_{Rmin}}}$$

$$P_{R_{min}} = 0.55125 \cdot 41.51^3 + 4.91 \cdot 10^6 \frac{1}{41.51} = 157.7 \ \mathrm{kW}$$

d2) $V_{F_{TRmin}}$

$$F_G = F_L = \frac{1}{2}\rho V^2 C_L A \quad \Longrightarrow \quad V^2 = \frac{2F_G}{C_L \rho A}$$

$$V = \sqrt{\frac{2mg}{C_L \rho A}} \; ; \quad C_L = 0.745356 \quad iz \quad c)$$

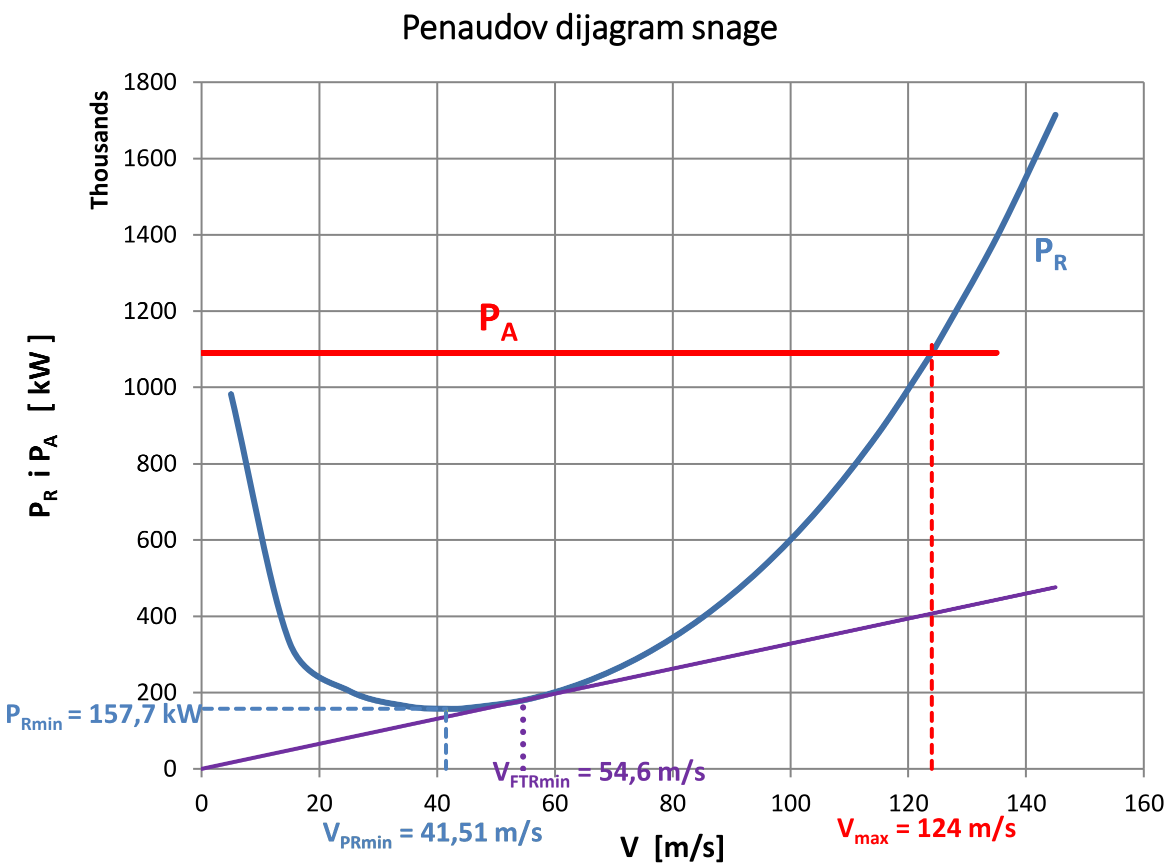
$$V = \sqrt{\frac{2 \cdot 5000 \cdot 9.81}{0.745356 \cdot 1.225 \cdot 36}} = 54.6 \;\; \mathrm{m/s}$$

$$\boxed{V_{F_{TRmin}} = 54.6 \quad \mathrm{m/s}}$$

d3) $V_{max} =?$ ako je $P_A = 1091 \text{ kW}$

$$V_{max} = 124 \;\; \mathrm{m/s} \qquad \text{iz dijagrama}$$

d4) raspon brzina u I režimu leta $V = (42 \ldots..124) \;\; \mathrm{m/s}$ iz dijagrama



4.5. Za laki školski zrakoplov na mlazni pogon mase 1540 kg i površine krila 15.6 m^2 zadana je polara u obliku tablice:

C_L	0.06	0.09	0.167	0.395	0.7	0.848
C_D	0.0186	0.0182	0.0201	0.0282	0.0516	0.0688

Nacrtaj krivulju potrebnog potiska i snage (Pénaud) za nadmorske visine 0 i 4000 m za horizontalni let.

$$F_{TR} = F_D = C_D \frac{1}{2} \rho V^2 A$$

$$F_G = F_L = C_L \frac{1}{2} \rho V^2 A \quad \Rightarrow \quad \frac{1}{2} \rho V^2 A = \frac{F_G}{C_L}$$

$$F_{TR} = C_D \frac{F_G}{C_L} = \frac{C_D}{C_L} F_G = \frac{C_D}{C_L} mg$$

$$F_{TR} = \frac{C_D}{C_L} \cdot 1540 \cdot 9.81 = 15107.4 \cdot \frac{C_D}{C_L} \qquad (1)$$

$$V = \sqrt{\frac{2F_G}{C_L \rho A}} = \sqrt{\frac{2mg}{C_L \rho A}}$$

$$H = 0\,m \quad \Rightarrow \quad \rho = 1.225 \text{ kg/m}^3$$

$$V = \sqrt{\frac{2 \cdot 1540 \cdot 9.81}{1.225 \cdot 15.6 C_L}} = 39.8 \sqrt{\frac{1}{C_L}} \qquad (2)$$

$$H = 4000\,m \quad \Rightarrow \quad \rho = 0.8194 \text{ kg/m}^3$$

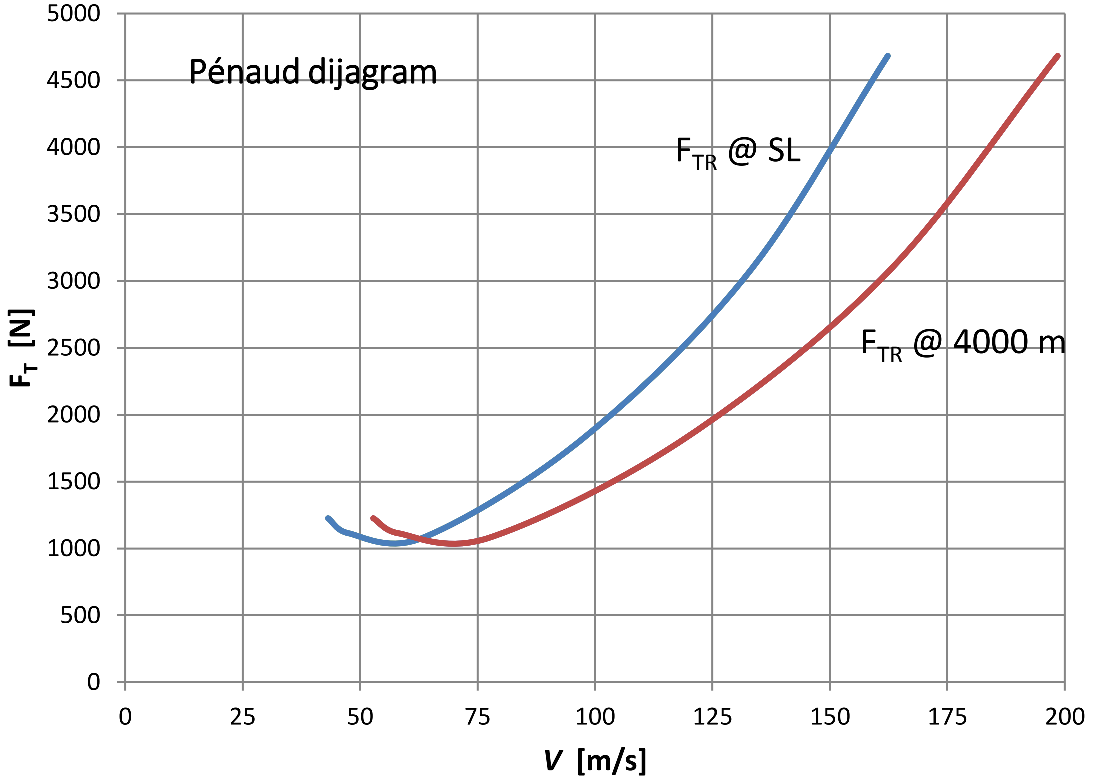
$$V = \sqrt{\frac{2 \cdot 1540 \cdot 9.81}{0.8194 \cdot 15.6 C_L}} = 48.6 \sqrt{\frac{1}{C_L}} \qquad (3)$$

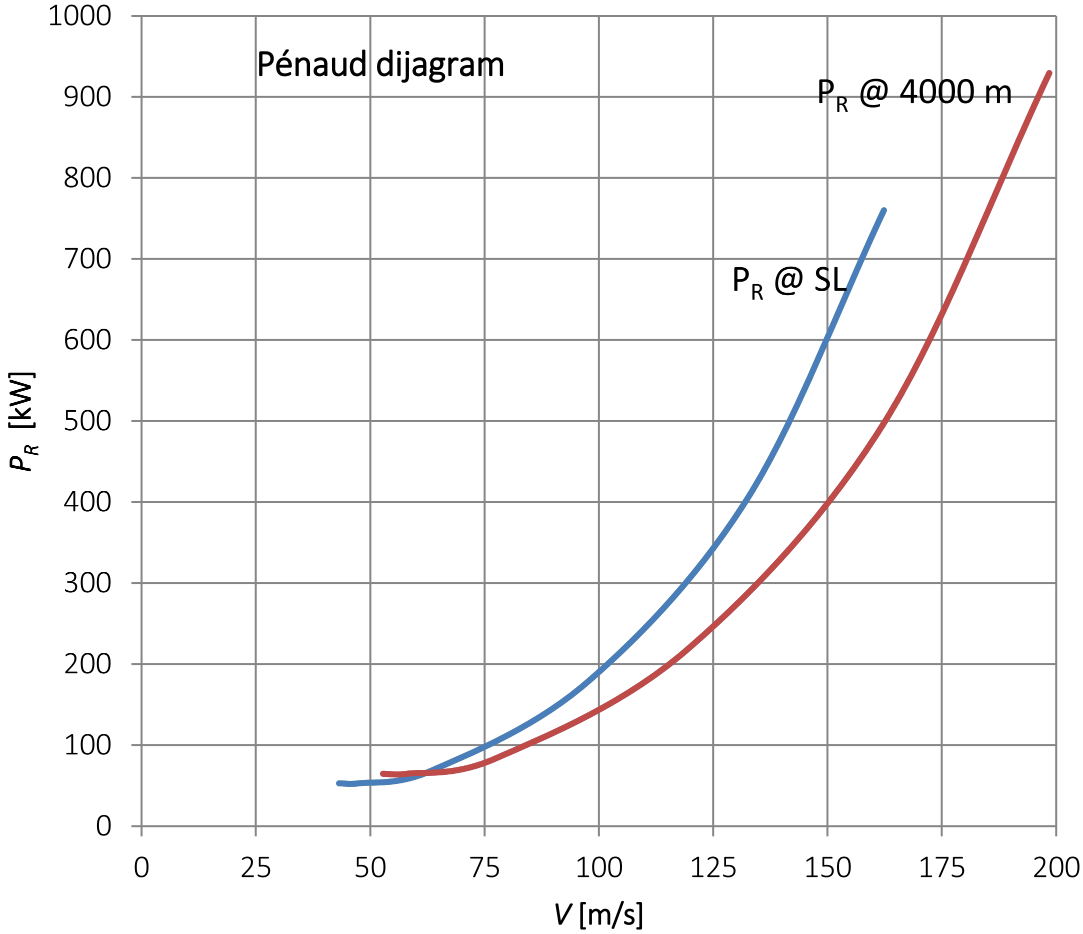
$$P_R = F_{TR} \cdot V$$

$$H = 0\,m \quad \Rightarrow \quad P_R = 15107 \cdot \frac{C_D}{C_L} \cdot 39.8 \sqrt{\frac{1}{C_L}} \quad \Rightarrow \quad P_R = 600716 \frac{C_D}{{C_L}^{3/2}} \qquad (4)$$

$$H = 4000\,m \quad \Rightarrow \quad P_R = 15107 \cdot \frac{C_D}{C_L} \cdot 48.6 \sqrt{\frac{1}{C_L}} \quad \Rightarrow \quad P_R = 734522 \frac{C_D}{{C_L}^{3/2}} \qquad (5)$$

C_L	C_D	F_{TR}	V @ 0 m	V @ 4000 m	P_R @ 0 m	P_R @ 4000 m	C_L/C_D	$C_D/C_L^{3/2}$
0,06	0,019	4683	162	198,5	760	930	3,226	1,266
0,09	0,018	3055	132,5	162	405	495	4,945	0,674
0,167	0,020	1818	97	119	177	216	8,308	0,295
0,395	0,028	1079	63	77	68	83	14,007	0,114
0,7	0,052	1114	47,5	58	53	65	13,566	0,088
0,848	0,069	1226	43	53	53	64,7	12,326	0,088




Pénaud dijagram
P_R @ 4000 m
P_R @ SL
P_R [kW]
1000
900
800
700
600
500
400
300
200
100
0
0
25
50
75
100
125
150
175
200
V [m/s]

4.6. Određivanje maksimalne brzine za zrakoplove Cessna Skylane i Cessna Citation 3 na razini mora.

Cessna Skylane

$1 \text{ klipni motor } P_{mot} = 171.5 \ \ \text{kW}$

$\eta = 0.8$

$P_A = \eta \cdot P_{mot} = 0.8 \cdot 171.5 = 137.2 \ \text{kW}$

Cessna Citation 3

$2 \text{ turbofan motora } \quad F_{T1} = 16236 \ \text{N}$

$F_{TA} = \ 2 \cdot F_{T1} = 2 \cdot 16236 = 32472 \ \text{N}$

$P_A = \ F_{TA} \cdot V = 32472 \cdot V$

$$\boldsymbol{P_R = P_A}$$

Cessna Skylane:

$$0.248\, V^3 + 0.947 \cdot 10^6 \frac{1}{V} = 137200$$

$$0.248\, V^4 = 137200V - 0.947 \cdot 10^6$$

$$V^4 = 553407V - 3.819 \cdot 10^6$$

$$V = \sqrt[4]{553407 \cdot V - 3.819 \cdot 10^6}$$

V_i (m/s)	V_{i+1} (m/s)
100	84.7
84.7	81
81	80
80	80

$$\boxed{\boldsymbol{V_{max} \ = 80 \ \mathrm{m/s} = 288 \ \mathrm{km/h}}}$$

Cessna Citation 3:

$$0.361\, V^3 + 19 \cdot 10^6 \frac{1}{V} = 32472V$$

$$0.361\, V^4 - 32472V^2 + 19 \cdot 10^6 = \ 0$$

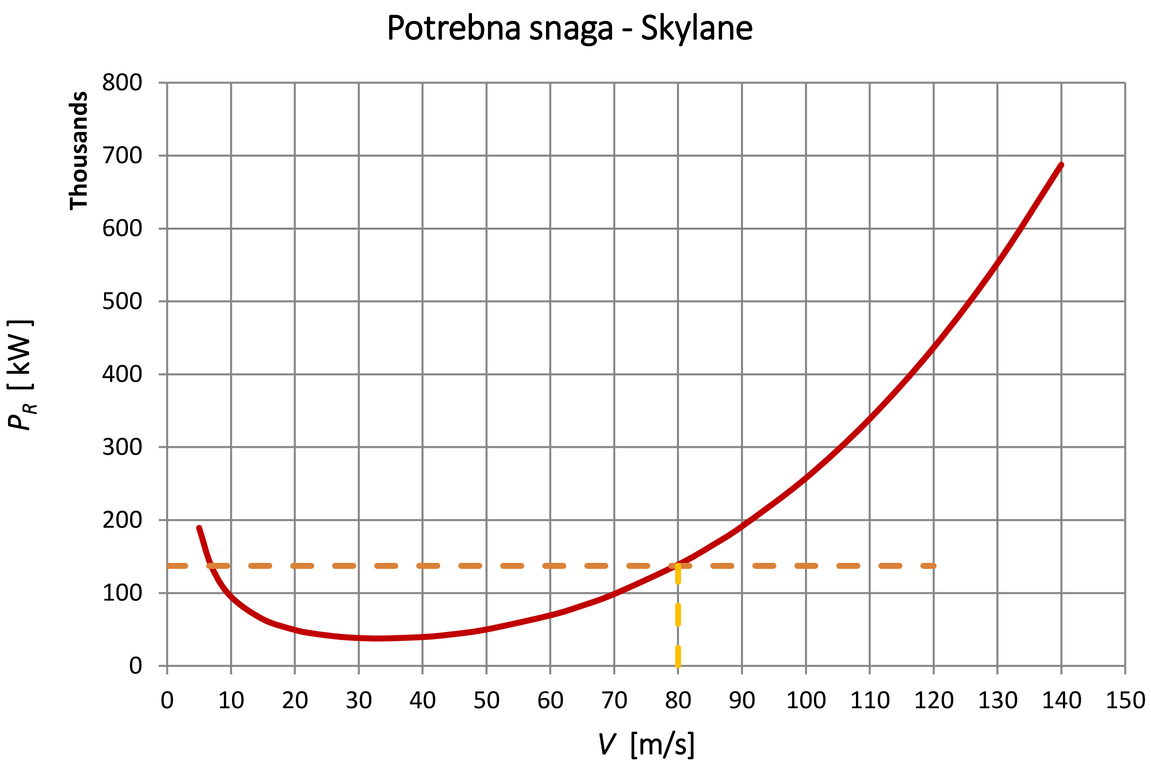
$${V_{1,2}}^2 = \frac{32472 \pm \sqrt{1.0544 \cdot 10^9 - 27.436 \cdot 10^6}}{2 \cdot 0.361}$$

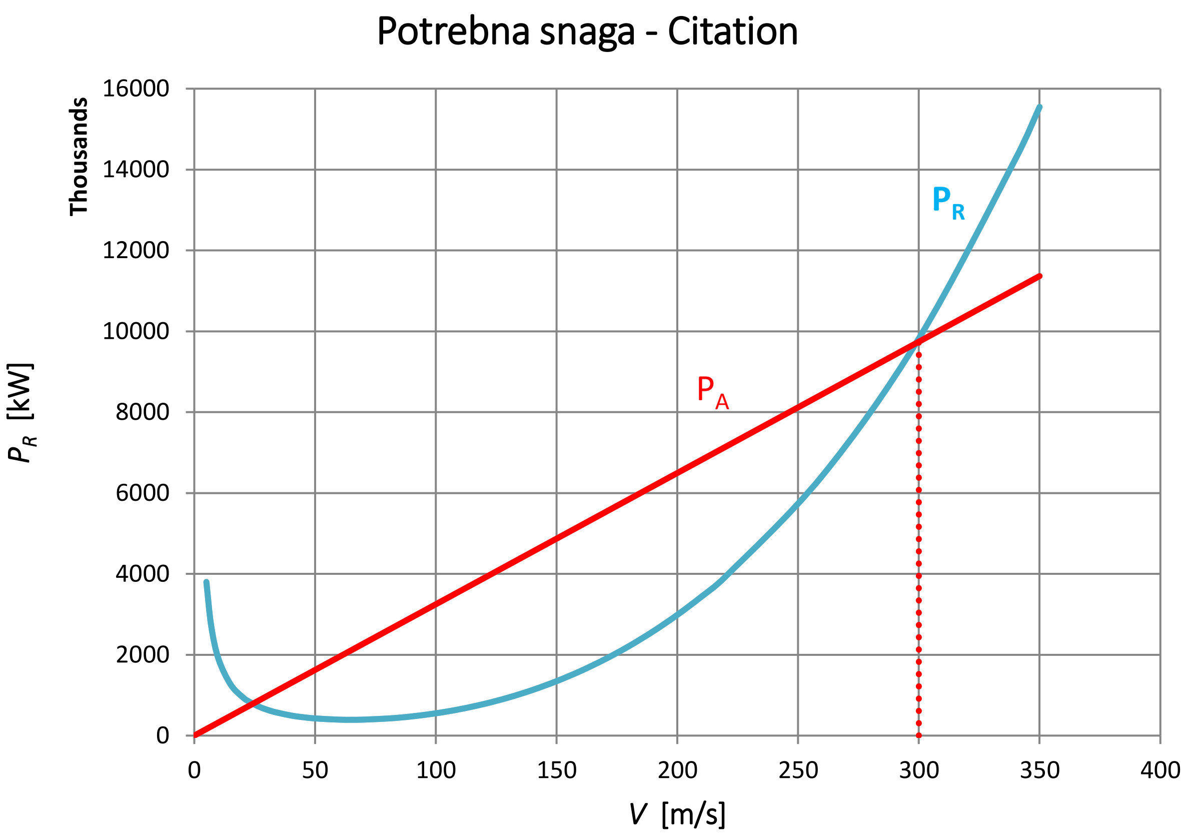
$${V_{1,2}}^2 = \frac{32472 \pm 32046}{0.722}$$

$${V_1}^2 = 89360.1$$

$$V_1 = 299\, \mathrm{m/s} = 1076 \ \mathrm{km/h}$$

$$\boxed{\boldsymbol{V_{max} \ = 299 \ \mathrm{m/s} = 1076 \ \mathrm{km/h}}}$$


Potrebna snaga - Skylane
Thousands
P_R [kW]
800
700
600
500
400
300
200
100
0
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150
V [m/s]


Potrebna snaga - Citation
Thousands
P_R [kW]
16000
14000
12000
10000
8000
6000
4000
2000
0
P_R
P_A
0 50 100 150 200 250 300 350 400
V [m/s]

4.7. Potrebno je odrediti potrebnu snagu za zrakoplov Cessna Skylane u zavisnosti o brzini leta i mase zrakoplova od 1000, 1400 i 1800 kg u uvjetima ISA/SL.

	b	*A*	*m*	C_{D0}	*e*	*AR*
Cessna Skylane	10.9	16.2	1338.1	0.025	0.8	7.3

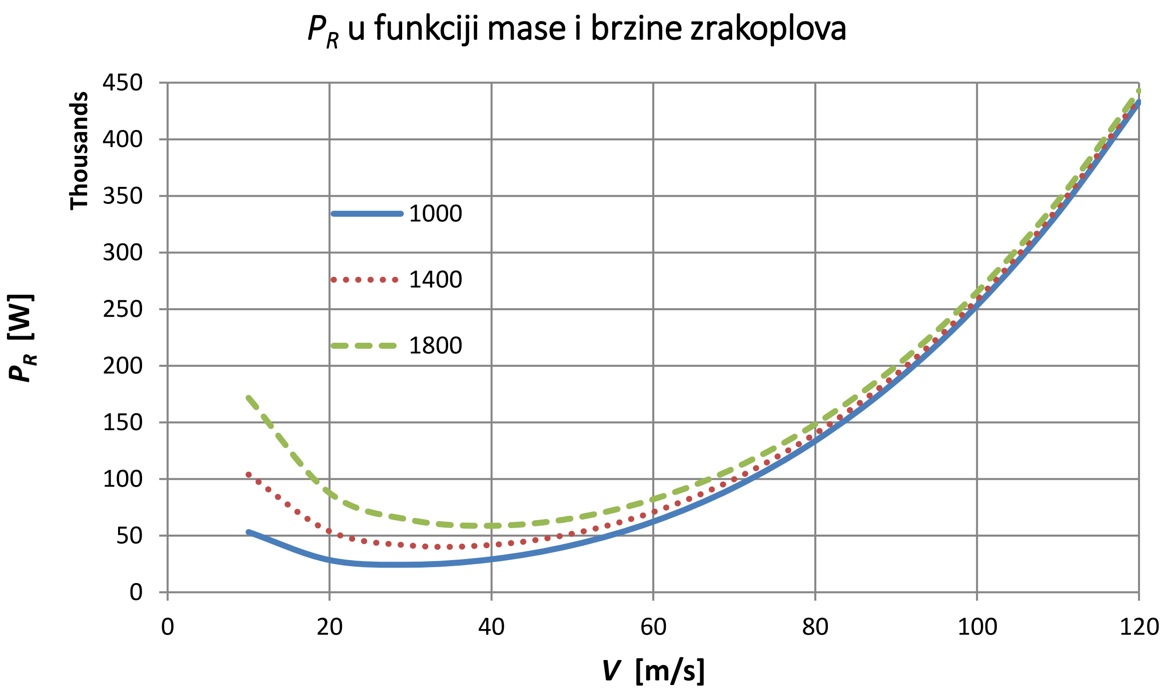
$$P_R = F_D \cdot V$$

$$F_{TR} = F_D = \frac{1}{2} C_{D0}\, \rho A \cdot V^2 + \frac{2{F_G}^2}{\pi e A R \rho A} \frac{1}{V^2}$$

$$P_R = \frac{1}{2} C_{D0}\, \rho A \cdot V^3 + \frac{2{F_G}^2}{\pi e A R \rho A} \frac{1}{V} = \frac{1}{2} \cdot 0.025 \cdot 1.225 \cdot 16.2 V^3 + \frac{2 \cdot 9.81^2}{\pi \cdot 0.8 \cdot 7.3 \cdot 1.225 \cdot 16.2} \frac{m^2}{V}$$

$$P_R = 0.248 V^3 + 0.529 \frac{m^2}{V}$$

m [kg]	P_R [W]
1000	$0.248V^3 + 529 \cdot 10^3 \frac{1}{V}$
1400	$0.248V^3 + 1037 \cdot 10^3 \frac{1}{V}$
1800	$0.248V^3 + 1714 \cdot 10^3 \frac{1}{V}$



Potrebna snaga za veću masu zrakoplova troši se za svladavanje induciranog otpora.

4.8. Potrebno je odrediti potrebnu snagu za zrakoplov Cessna Citation 3 u zavisnosti o brzini leta i u uvjetima ISA/SL, s obzirom na povećani parazitni otpor uslijed izvučenog podvozja $\Delta \boldsymbol{C_{D0}} = \mathbf{0.01}$.

	b	*A*	*m*	*C_{D0}*	*e*	*AR*
Cessna Citation 3	16.2	29.5	8987.9	0.02	0.81	8.9

Rješenje:

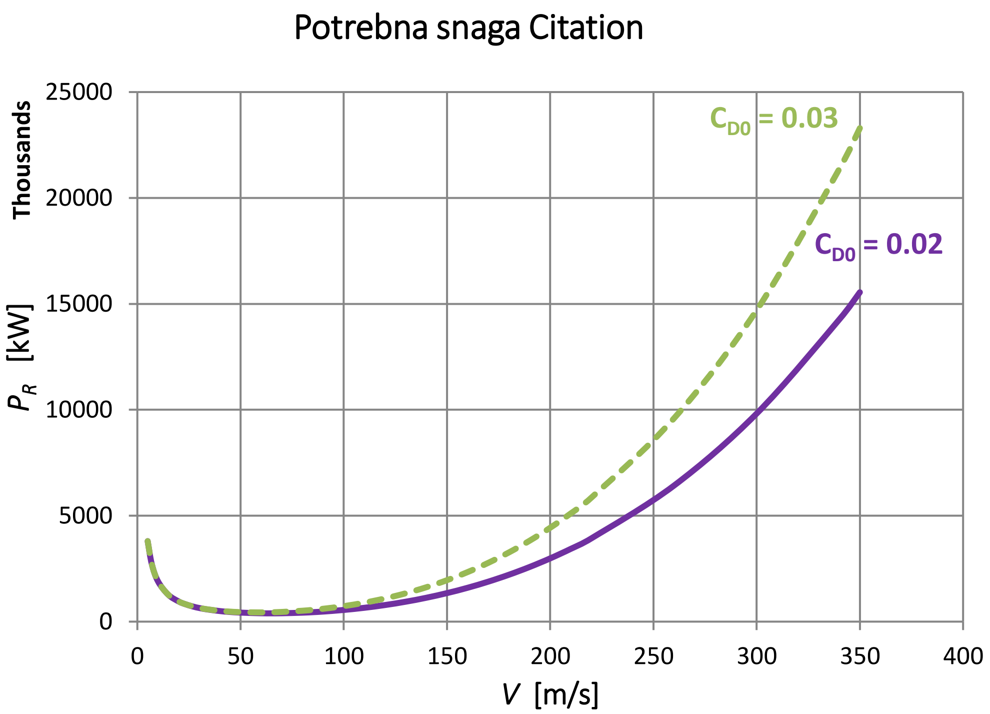
$$F_{TR} = \frac{1}{2} C_{D0}\, \rho A \cdot V^2 + \frac{2{F_G}^2}{\pi e A R \rho A} \frac{1}{V^2}$$

$$P_R = F_{TR} V = \frac{1}{2} C_{D0}\, \rho A \cdot V^3 + \frac{2{F_G}^2}{\pi e A R \rho A} \frac{1}{V} =$$

$$= \frac{1}{2} C_{D0}\, 1.225 \cdot 29.5 \cdot V^3 + \frac{2 \cdot (8987.9 \cdot 9.81)^2}{\pi \cdot 0.81 \cdot 8.9 \cdot 1.225 \cdot 29.5} \frac{1}{V}$$

$$= 18.069 \cdot C_{D0}\, V^3 + 19 \cdot 10^6 \frac{1}{V}$$

$\boldsymbol{C_{D0}}$	$\boldsymbol{P_R}$
0.02	$\boldsymbol{P_R = 0.3614\, V^3 + 19 \cdot 10^6 \frac{1}{V}}$
$\mathbf{0.03}$	$\boldsymbol{P_{R_{LG}} = 0.5421\, V^3 + 19 \cdot 10^6 \frac{1}{V}}$



4.9. Promatra se zrakoplov Beechcraft Bonanza V35B čije su karakteristike:

- jednomotorni zrakoplov s V repom
- masa zrakoplova m = 1360 kg
- površina krila A = 16.8 m2
- aspektni odnos AR = 6.2
- Oswaldov koeficijent e = 0.91
- Koeficijent parazitnog otpora C_{D0} = 0.027
- Snaga motora P_{mot} = 256 kW
- Korisnost propelera η = 0.85

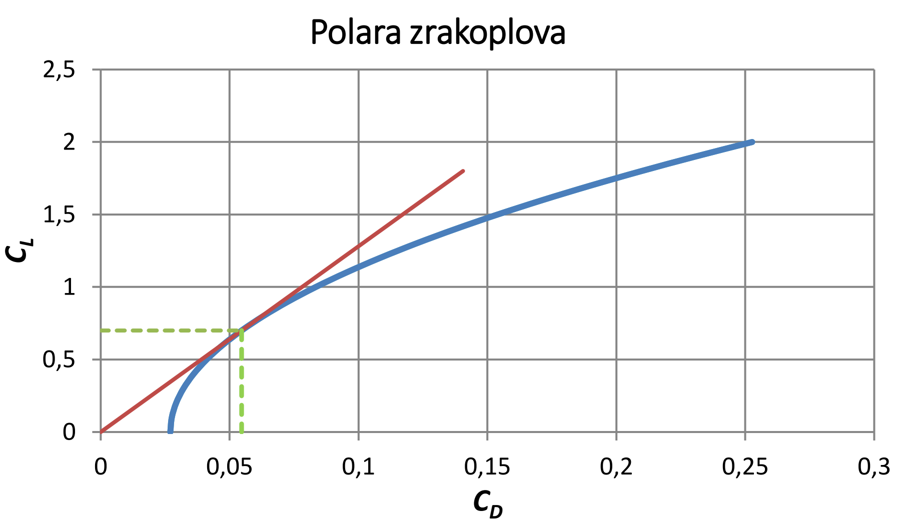
Odrediti analitički i grafički:

a) polaru zrakoplova
b) ovisnost potrebne snage o brzini
c) potrebnu snagu za let brzinom 50 m/s
d) maksimalnu finesu zrakoplova i odgovarajuću snagu i brzinu
e) podatke za najbolju istrajnost leta
f) maksimalnu brzinu zrakoplova

Rješenje:

a) $C_D = C_{D0} + {C_L}^2/_{\pi e AR}$

$$C_D = 0.027 + \frac{{C_L}^2}{\pi \cdot 0.91 \cdot 6.2} = 0.027 + 0.0564{C_L}^2$$



b)

$$P_R = F_{TR} \cdot V = F_D \cdot V = C_D \frac{1}{2} \rho V^2 A \cdot V = C_D \frac{1}{2} \rho V^3 A$$

$$P_R = \left(0.027 + 0.0564 {C_L}^2\right) \frac{1}{2} \rho V^3 A$$

$$C_L = \frac{2mg}{\rho V^2 A}$$

$$P_R = \left(0.027 + 0.0564 \frac{4{F_G}^2}{\rho^2 V^4 A^2}\right) \frac{1}{2} \rho V^3 A$$

$$P_R = 0.0135 \rho A V^3 + 0.1128 \frac{{F_G}^2}{\rho A V}$$

$$P_R = 0.0135 \cdot 1.225 \cdot 16.8 V^3 + 0.1128 \frac{(1360 \cdot 9.81)^2}{1.225 \cdot 16.8 V}$$

$$P_R = 0.27783 V^3 + 975617 \frac{1}{V}$$

c)

$$P_R = 0.27783 V^3 + 975617 \frac{1}{V}$$

$$P_R = 0.27783 \cdot 50^3 + 975617 \frac{1}{50} = 54.2 \ \text{kW}$$

d)

$$C_{D0} = C_{Di}$$

$$0.027 = 0.0564 {C_L}^2$$

$$C_L = \sqrt{\frac{0.027}{0.0564}} = 0.692$$

$$C_D = 2 C_{D0} = 2 \cdot 0.027 = 0.054$$

$$f = \frac{C_L}{C_D} = \frac{0.692}{0.054} = 12.81$$

$$V = \sqrt{\frac{2mg}{C_L \rho A}} = \sqrt{\frac{2 \cdot 1360 \cdot 9.81}{0.692 \cdot 1.225 \cdot 16.8}} = 43.3 \ \text{m/s}$$

$$P_R = \frac{C_D}{{C_L}^{3/2}} \sqrt{\frac{2{F_G}^3}{\rho A}} = \frac{0.054}{0.692^{3/2}} \sqrt{\frac{2(1360 \cdot 9.81)^3}{1.225 \cdot 16.8}} = 45.1 \ \text{kW}$$

e)

$$C_{D0} = \frac{1}{3} C_{Di}$$

$$0.027 = \frac{1}{3} 0.0564 {C_L}^2$$

$$C_L = \sqrt{\frac{3 \cdot 0.027}{0.0564}} = 1.198$$

$$C_D = 0.027 + 0.0564 \cdot 1.198^2 = 0.108$$

$$V = \sqrt{\frac{2mg}{C_L \, \rho \, A}} = \sqrt{\frac{2 \cdot 1360 \cdot 9.81}{1.198 \cdot 1.225 \cdot 16.8}} = 32.9 \ \text{m/s}$$

$$P_R = \frac{C_D}{{C_L}^{3/2}} \sqrt{\frac{2{F_G}^3}{\rho \, A}} = \frac{0.108}{0.1.198^{3/2}} \sqrt{\frac{2(1360 \cdot 9.81)^3}{1.225 \cdot \ 16.8}} = 39.6 \ \text{kW}$$

f)

$$P_R = P_A = \eta \cdot P$$

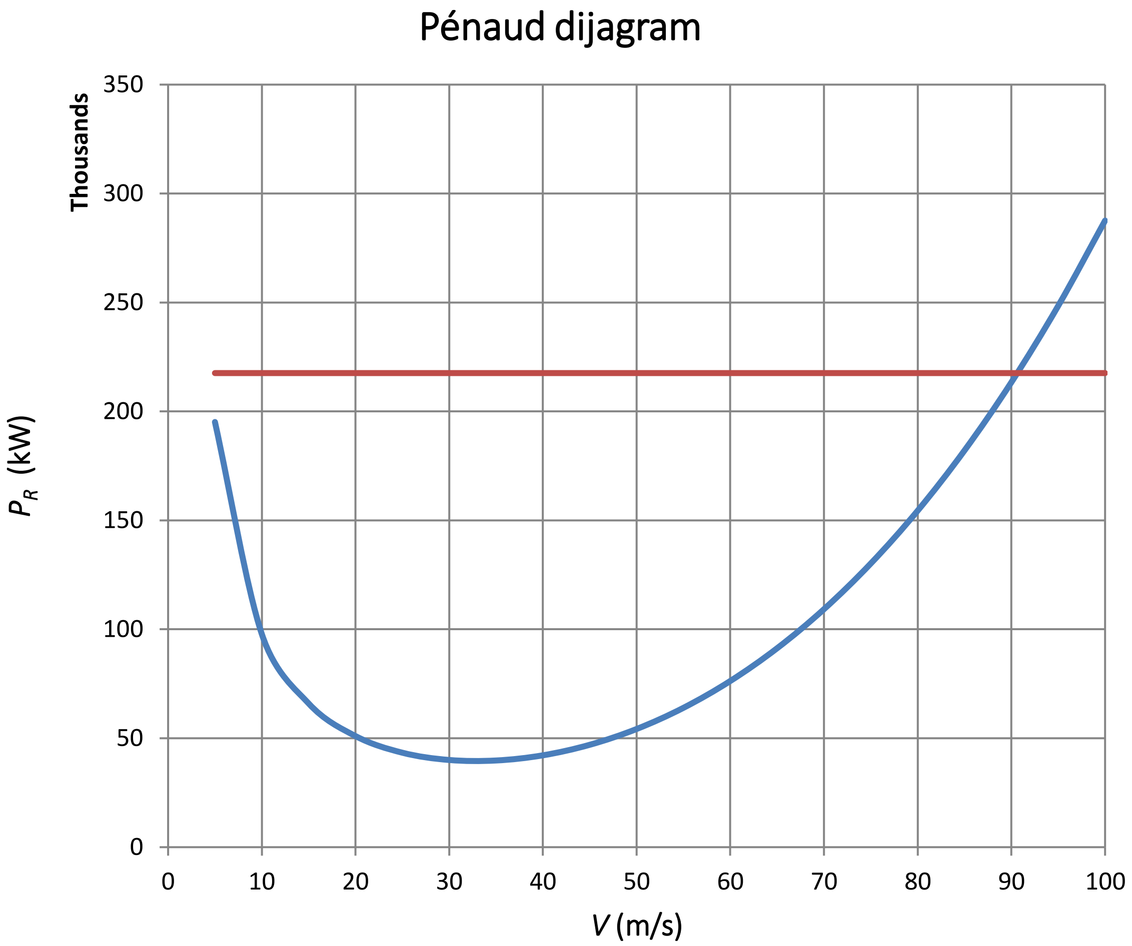
$$0.27783 V^3 + 975617 \frac{1}{V} = 0.85 \cdot 256000 \ / \cdot V$$

$$0.27783 V^4 = 217600 V - 975617$$

$$V = \sqrt[4]{783213 \cdot V - 3511561}$$

V_i (m/s)	V_{i+1} (m/s)
100	93
93	91.25
91.25	90.79
90.79	90.67

$$V_{max} = 90.6 \ \text{m/s}$$


Pénaud dijagram
Thousands
P_R (kW)
350
300
250
200
150
100
50
0
0
10
20
30
40
50
60
70
80
90
100
V (m/s)

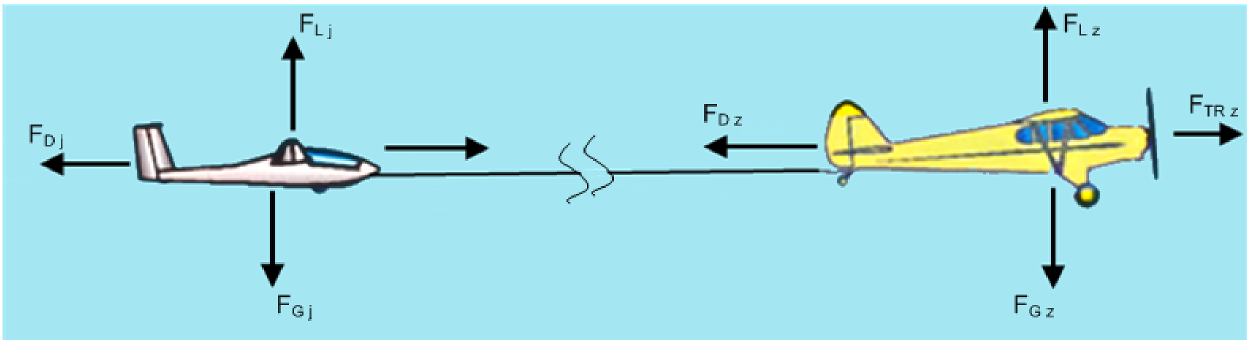
4.10. Zrakoplov vuče na tankom čeličnom užetu jedrilicu. Zrakoplov ima masu 4650 kg i površinu krila $\boldsymbol{A} = \mathbf{37.6\ m^2}$, polara je zadana analitički:

$$\boldsymbol{C_D = 0.024 + 0.064{C_L}^2}$$

Jedrilica ima masu 1650 kg i površinu krila 40.8 m^2, a polara je također dana analitički:

$$\boldsymbol{C_D = 0.02 + 0.041{C_L}^2}$$

Odrediti analitički izraz za potrebnu vučnu silu i za potrebnu snagu za horizontalni let aerozaprege. Odrediti minimalnu vrijednost potrebne vučne sile i odgovarajuću brzinu. Odrediti silu u užetu u tom slučaju. Otpore u užetu zanemariti.



Rješenje:

$$F_{TR} = F_{D\,z} + F_{D\,j} = C_{D\,z}\frac{1}{2}\rho V^2 A_z + C_{D\,j}\frac{1}{2}\rho V^2 A_j$$

$$F_{TR} = \frac{1}{2}\left(0.024 + 0.064{C_{L\,z}}^2\right)\rho V^2 A_z + \frac{1}{2}\left(0.02 + 0.041{C_{L\,j}}^2\right)\rho V^2 A_j$$

$$F_{TR} = \left(0.012A_z + 0.01A_j\right)\rho V^2 + \left(0.032{C_{L\,z}}^2 A_z + 0.0205{C_{L\,j}}^2 A_j\right)\rho V^2$$

$$F_{G\,z} = C_{L\,z}\frac{1}{2}\rho V^2 A_z \quad \Rightarrow \quad C_{L\,z} = \frac{2m_z g}{\rho V^2 A_z}$$

$$F_{G\,j} = C_{L\,j}\frac{1}{2}\rho V^2 A_j \quad \Rightarrow \quad C_{L\,j} = \frac{2m_j g}{\rho V^2 A_j}$$

$$F_{TR} = \left(0.012A_z + 0.01A_j\right)\rho V^2 + \left(0.032\frac{4(m_z g)^2}{\rho^2 V^4 {A_z}^2}A_z + 0.0205\frac{4\left(m_j g\right)^2}{\rho^2 V^4 {A_j}^2}A_j\right)\rho V^2$$

$$F_{TR} = \left(0.012A_z + 0.01A_j\right)\rho V^2 + \left(0.128\frac{(m_z g)^2}{A_z} + 0.082\frac{\left(m_j g\right)^2}{A_j}\right)\frac{1}{\rho V^2}$$

$$F_{TR} = (0.012\cdot 37.6 + 0.01\cdot 40.8)\rho V^2 + \left(0.128\frac{(4650\cdot 9.81)^2}{37.6} + 0.082\frac{(1650\cdot 9.81)^2}{40.8}\right)\frac{1}{\rho V^2}$$

$$F_{TR} = 0.8592\rho V^2 + 7610370.2\ \frac{1}{\rho V^2}$$

$H = 0$ m $\quad \Rightarrow \quad \rho = 1.225$ kg/m^3

$$F_{TR} = 1.05252V^2 + 6212547.1\ \frac{1}{V^2}$$

$$P_R = F_{TR} \cdot V = 1.05252V^3 + 6212547.1\ \frac{1}{V}$$

$$\frac{dF_{TR}}{dV} = 2.10504 \cdot V - 12425094.2\frac{1}{V^3} = 0 \qquad /\cdot V^3$$

$$2.10504 \cdot V^4 - 12425094.2 = 0$$

$$V^4 = 5902545.4$$

$$V = 49.3\ \ \mathrm{m/s}$$

$$F_{TR_{min}} = 1.05252 \cdot 49.3^2 + 6212547.1\frac{1}{49.3^2} = 5114.2\ \ \mathrm{N}$$

$$F_{T_{uže}} = C_{D\,j}\frac{1}{2}\ \rho V^2 A_j = \frac{1}{2}\left(0.02 + 0.041{C_{L\,j}}^2\right)\ \rho V^2 A_j$$

$$C_{L\,j} = \frac{2m_j g}{\rho V^2 A_j} = \frac{2 \cdot 1650 \cdot 9.81}{1.225 \cdot 49.3^2 \cdot 40.8} = 0.2665$$

$$F_{T_{uže}} = \frac{1}{2}(0.02 + 0.041 \cdot 0.2665^2)\ \cdot 1.225 \cdot 49.3^2 \cdot 40.8 = 1391.6\ \ \mathrm{N}$$

5. JEDNOLIKO PENJANJE I SPUŠTANJE ZRAKOPLOVA

5.1. Za zrakoplove Cessna Skylane i Cessna Citation 3 odredi ovisnost brzine uzdizanja o brzini i maksimalnu brzinu uzdizanja!

$$\boldsymbol{R/C = \frac{\Delta P}{F_G} = \frac{P_A - P_R}{F_G}}$$

	A	*m*	C_{D0}	*e*	*AR*	P_A	P_R
Cessna Skylane	16.2	1338.1	0.025	0.8	7.3	137200	$0.248\,V^3 + 0.947 \cdot 10^6 \frac{1}{V}$
Cessna Citation 3	29.5	8988	0.02	0.81	8.9	$32472 \cdot V$	$0.361\,V^3 + 19 \cdot 10^6 \frac{1}{V}$

SKYLANE

$$P_A = 137\,200 \;\; \mathrm{W}$$

$$P_R = 0.248\,V^3 + 0.947 \cdot 10^6 \frac{1}{V}$$

$$R/C = \frac{137\,200 - 0.248\,V^3 - 0.947 \cdot 10^6 \frac{1}{V}}{1338.1 \cdot 9.81}$$

$$\frac{d(R/C)}{dV} = -0.744V^2 + 0.947 \cdot 10^6 \frac{1}{V^2} = 0$$

$$0.744V^4 = 0.947 \cdot 10^6$$

$V_{(R/C)_{max}} = 33.6 \;\; \mathrm{m}/s$ ... jednaka brzini za minimalnu potrebnu snagu V_{PRmin}

$$(R/C)_{max} = \frac{137\,200 - 0.248\;33.6^3 - 0.947 \cdot 10^6 \frac{1}{33.6}}{1338.1 \cdot 9.81}$$

$$(R/C)_{max} = 7.6 \;\; \mathrm{m/s}$$

CITATION

$$P_A = 32\,472 \cdot V$$

$$P_R = 0.361\,V^3 + 19 \cdot 10^6 \frac{1}{V}$$

$$R/C = \frac{32\,472 \cdot V - 0.361\,V^3 - 19 \cdot 10^6 \frac{1}{V}}{8987.9 \cdot 9.81}$$

$$\frac{d(R/C)}{dV} = 32\,472 - 1.083 \cdot V^2 + 19 \cdot 10^6 \frac{1}{V^2} = 0$$

$$1.083 \cdot V^4 - 32\,472 \cdot V^2 - 19 \cdot 10^6 = 0$$

$${V_{1,2}}^2 = \frac{32472 \pm \sqrt{1.0544 \cdot 10^9 + 0.0823 \cdot 10^9}}{2 \cdot 1.083}$$

$${V_{1,2}}^2 = \frac{32472 \pm 33716}{2.166}$$

$$V_{(R/C)_{max}} = 174.8 \quad \text{m/s}$$

$$(R/C)_{max} = \frac{32\,472 \cdot 174.8 - 0.361\,174.8^3 - 19 \cdot 10^6 \frac{1}{174.8}}{8987.9 \cdot 9.81} = 41.3 \ \text{m/s}$$

5.2. Odredi maksimalni kut penjanja za Cessna Skylane i Cessna Citation 3.

$\boldsymbol{SKYLANE}$

$$R/C = \frac{137\,200 - 0.248\,V^3 - 0.947 \cdot 10^6 \frac{1}{V}}{1338.1 \cdot 9.81} = 10.452 - 1.889 \cdot 10^{-5} V^3 - 72.143 \frac{1}{V}$$

$$\frac{R/C}{V} = \frac{10.452}{V} - 1.889 \cdot 10^{-5} V^2 - 72.143 \frac{1}{V^2}$$

$$\frac{d\left(\frac{R/C}{V}\right)}{dV} = -\frac{10.452}{V^2} - 3.778 \cdot 10^{-5} V + \frac{144.286}{V^3} = 0$$

$$3.778 \cdot 10^{-5} V^4 + 10.452V - 144.286 = 0$$

$$V^4 = 3.819 \cdot 10^6 - 2.767 \cdot 10^5 \cdot V$$

$$V = \sqrt[4]{3.819 \cdot 10^6 - 2.767 \cdot 10^5 \cdot V}$$

i	V_i	V_{i+1}
1	10	32.03
2	12	26.58
3	13	21.72
4	13.5	17.03
5	13.7	13.03
6	13.67	13.88
7	13.68	13.62

$$\boxed{V_{\theta_{max}} = 13.7 \text{ m/s}}$$

Ovako teorijski izračunata brzina za najveći kut penjanja je manja od brzine sloma uzgona u praksi.

$$(R/C)_{\theta_{max}} = 10.452 - 1.889 \cdot 10^{-5} \cdot 13.7^3 - 72.143 \frac{1}{13.7} = 5.14 \text{ m/s}$$

$$\theta_{max} = \arcsin\left(\frac{5.14}{13.7}\right) = 22.02°$$

CITATION

$$R/C = \frac{32\,472 \cdot V - 0.361\,V^3 - 19 \cdot 10^6 \frac{1}{V}}{8987.9 \cdot 9.81} = 0.3683 \cdot V - 4.094 \cdot 10^{-6}\,V^3 - 215.5\frac{1}{V}$$

$$\frac{R/C}{V} = 0.3683 - 4.094 \cdot 10^{-6}\,V^2 - \frac{215.5}{V^2}$$

$$\frac{d(R/C)}{dV} = 0 \qquad \Rightarrow \qquad -8.188 \cdot 10^{-6} \cdot V + \frac{431}{V^3} = 0$$

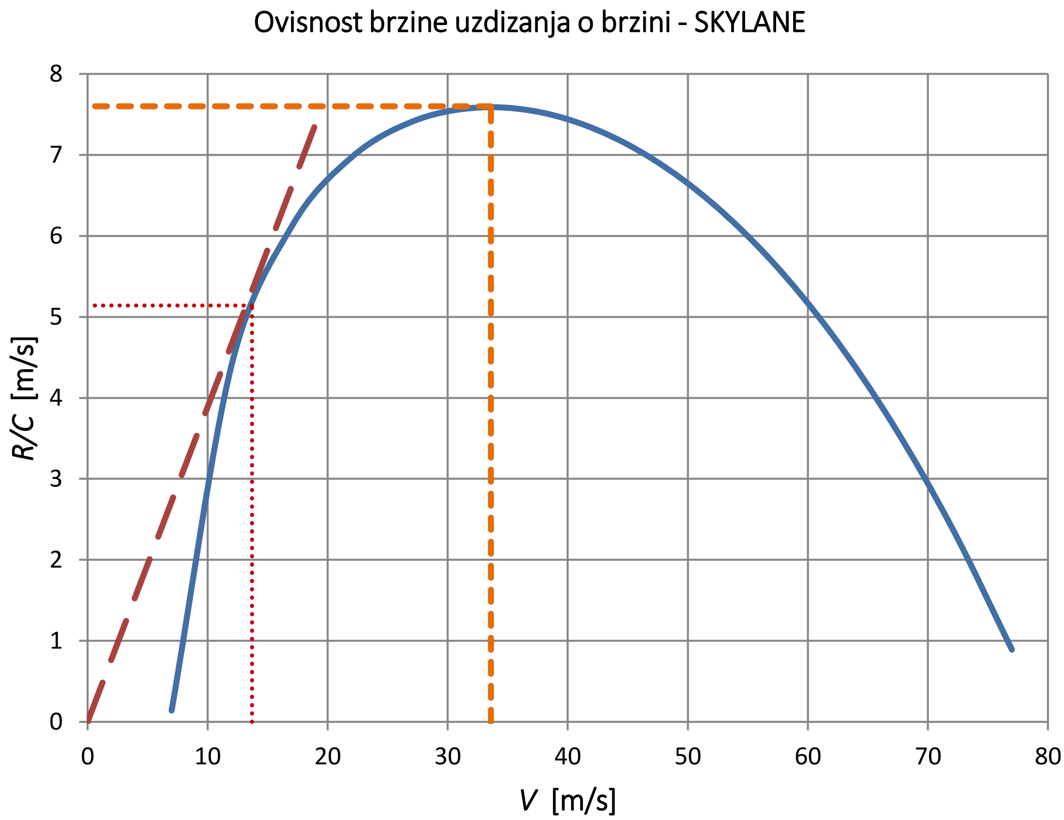
$$8.188 \cdot 10^{-6} \cdot V^4 - 431 = 0$$

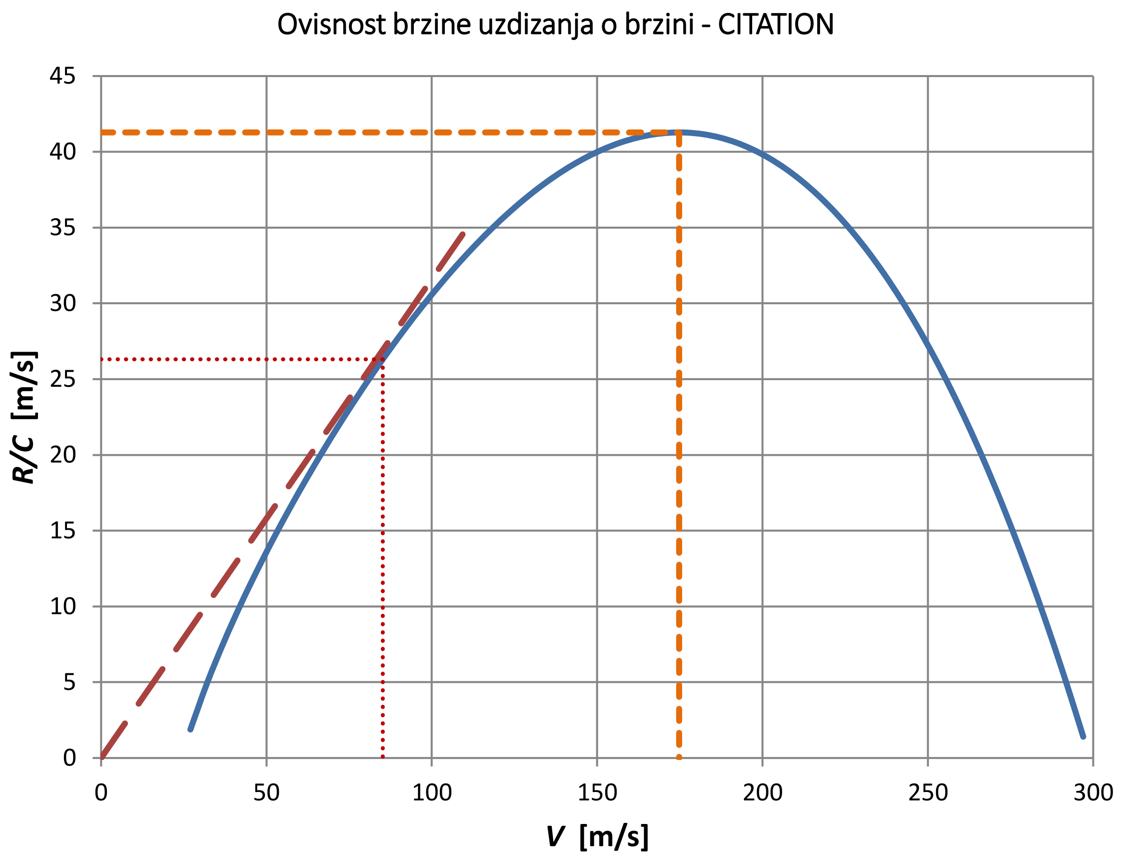
$$V = \sqrt[4]{\frac{431}{8.188 \cdot 10^{-6}}}$$

$$V_{\theta_{max}} = 85.2 \;\; m/s$$

$$(R/C)_{\theta_{max}} = 0.3683 \cdot 85.18 - 4.094 \cdot 10^{-6}\,85.18^3 - \frac{215.5}{85.18} = 26.3 \;\; \mathrm{m/s}$$

$$\theta_{max} = \mathrm{asin}\left(\frac{26.31}{85.18}\right) = 18°$$





5.3. Za zrakoplov mase 5000 kg, površine krila 36 m^2, s polarom $\boldsymbol{C_D = 0.025 + 0.045{C_L}^2}$. Odredi najkraće vrijeme penjanja od razine mora do 6000 m. Raspoloživa snaga motora je 1100 kW.

Rješenje:

$$dt = \frac{dh}{R/C} \quad \rightarrow \quad t = \int_{h_1}^{h_2} \frac{dh}{R/C} = \frac{h_2 - h_1}{\frac{(R/C)_1 + (R/C)_2}{2}} = \frac{2(h_2 - h_1)}{(R/C)_1 + (R/C)_2}$$

$$t_{min} = (R/C)_{max} = \frac{P_A - P_{R\,min}}{F_G} = \frac{P_A}{F_G} - \left(\frac{C_D}{{C_L}^{3/2}}\right)_{min} \sqrt{\frac{2F_G}{\rho\, A}}$$

$$\left(\frac{C_D}{{C_L}^{\frac{3}{2}}}\right)_{min} = \frac{d}{dC_L}\left(\frac{0.025 + 0.045{C_L}^2}{{C_L}^{\frac{3}{2}}}\right) = \frac{0.09C_L{C_L}^{\frac{3}{2}} - 1.5{C_L}^{\frac{1}{2}}\left(0.025 + 0.045{C_L}^2\right)}{{C_L}^3} = 0$$

$$0.0225{C_L}^2 = 0.0375$$

$$C_L = \sqrt{\frac{0.0375}{0.0225}} = 1.291$$

$$C_D = 0.025 + 0.045 \cdot 1.291^2 = 0.1$$

$$(R/C)_{max} = \frac{1100\,000}{5000 \cdot 9.81} - \frac{0.1}{1.291^{3/2}}\sqrt{\frac{2 \cdot 5000 \cdot 9.81}{\rho\ \cdot 36}} = 22.426 - 3.559\sqrt{\frac{1}{\rho}}$$

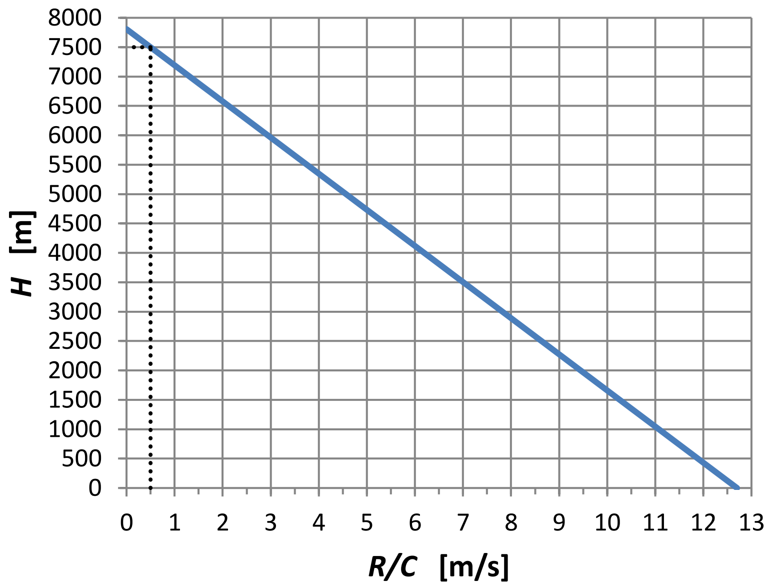
<table>
<tr><th>H [m]</th><th>ρ [kg/m3]</th><th>(R/C)_{max} [m/s]</th><th>t_{min} [s]</th></tr>
<tr><td>0</td><td>1,225</td><td>19,2</td><td rowspan="7">52,3 (0-1000 m)
52,7 (1000-2000 m)
53,2 (2000-3000 m)
53,8 (3000-4000 m)
54,4 (4000-5000 m)
55,1 (5000-6000 m)</td></tr>
<tr><td>1000</td><td>1,111628</td><td>19,1</td></tr>
<tr><td>2000</td><td>1,006464</td><td>18,9</td></tr>
<tr><td>3000</td><td>0,909086</td><td>18,7</td></tr>
<tr><td>4000</td><td>0,819085</td><td>18,5</td></tr>
<tr><td>5000</td><td>0,736064</td><td>18,3</td></tr>
<tr><td>6000</td><td>0,65964</td><td>18,0</td></tr>
<tr><td></td><td></td><td>∑</td><td>321.5 s = 5.4 min</td></tr>
</table>

$$t_{min} = \frac{2 \cdot 6000}{19.2 + 18.0} = 322.1\text{ s} = 5.4\text{ min}$$

5.4. Avion ima $(R/C)_{max}$ = 12.7 m/s na razini mora. R/C opada linearno s porastom visine. Praktični vrhunac leta iznosi 7500 m. Odredi apsolutni vrhunac leta.

Rješenje:

$$R/C = \frac{dh}{dt} \Rightarrow dt = \frac{1}{R/C}dh$$



$$\frac{H_{aps}}{12.7} = \frac{7500}{12.7 - 0.5} \quad \Rightarrow H_{aps} = \frac{7500 \cdot 12.7}{12.7 - 0.5} = 7807\text{ m}$$

5.5. Za zrakoplov mase 1200 kg i površine krila 21 m^2, s klipnim motorom nominalne snage 180 kW zadana je polara u analitičkom obliku: $\boldsymbol{C_D = 0.032 + 0.055{C_L}^2}$. Odredi maksimalne brzine uzdizanja na visinama 0, 1000 i 2000 m, te vrijeme penjanja do 1000 odnosno 2000 m.

$$(R/C)_{max} = \frac{P_A - P_{R\,min}}{F_G} = \frac{P_A}{F_G} - \frac{P_{R\,min}}{F_G}$$

$$(R/C)_{max} = \frac{P_A}{F_G} - \left(\frac{C_D}{{C_L}^{3/2}}\right)_{min} \sqrt{\frac{2mg}{\rho\, A}}$$

$$\frac{C_D}{{C_L}^{3/2}} = \frac{0.032 + 0.055{C_L}^2}{{C_L}^{3/2}}$$

$$\frac{d}{dC_L}\left(\frac{C_D}{{C_L}^{3/2}}\right)_{min} = \frac{0.110C_L{C_L}^{3/2} - \frac{3}{2}{C_L}^{1/2}\left(0.032 + 0.055{C_L}^2\right)}{{C_L}^3} = 0$$

$$0.110{C_L}^{5/2} - 0.048{C_L}^{1/2} - 0.0825{C_L}^{5/2} = 0 \qquad /\div {C_L}^{1/2}$$

$$0.0275{C_L}^2 = 0.048$$

$$C_L = \sqrt{\frac{0.048}{0.0275}} = 1.321$$

$$C_D = 0.032 + 0.055 \cdot 1.321^2 = 0.128$$

$$(R/C)_{max} = \frac{180\,000}{1200 \cdot 9.81} - \frac{0.128}{1.321^{3/2}}\sqrt{\frac{2 \cdot 1200 \cdot 9.81}{\rho\, \cdot 21}} = 15.2905 - 2.8223\sqrt{\frac{1}{\rho}}$$

H [m]	0	1000	2000
ρ [kg/m^3]	1.225	1.11166	1.00655
R/C [m/s]	12.74	12.61	12.48

$$\Delta t_{0-1000} = \frac{\Delta H}{(R/C)_{sr}} = \frac{1000 - 0}{\frac{(R/C)_0 + (R/C)_{1000}}{2}} = \frac{2 \cdot 1000}{12.74 + 12.61} = 78.9\ s = 1\text{ min } 19\text{ s}$$

$$\Delta t_{1000-2000} = \frac{\Delta H}{(R/C)_{sr}} = \frac{2000 - 1000}{\frac{(R/C)_{2000} + (R/C)_{1000}}{2}} = \frac{2 \cdot 1000}{12.48 + 12.61} = 79.7\text{ s} = 1\text{ min } 20\text{ s}$$

$$\Delta t_{0-2000} = 78.9 + 79.7 = 158.6\ s = 2\text{ min }\ 39\text{ s}$$

5.6. Za školski zrakoplov mase 1540 kg i površine krila 15.6 m^2 odredi brzinu uzdizanja u zavisnosti o brzini za visine 0 i 4000 m. Podaci za zrakoplov dani su tablično.

Tablica 1. Polara zrakoplova

C_L	0.06	0.09	0.167	0.395	0.7	0.848
C_D	0.0186	0.0182	0.0201	0.0282	0.0516	0.0688

Tablica 2. Raspoloživi potisak F_{T_A} $[N]$ s obzirom na visinu i brzinu leta

V[km/h] / *H* [m]	200	300	400	500
0	3800	3700	3600	3600
4000	2800	2700	2600	2600

Rješenje:

$F_{TR} = \frac{C_D}{C_L} F_G$ ... potisak potreban za horizontalni let

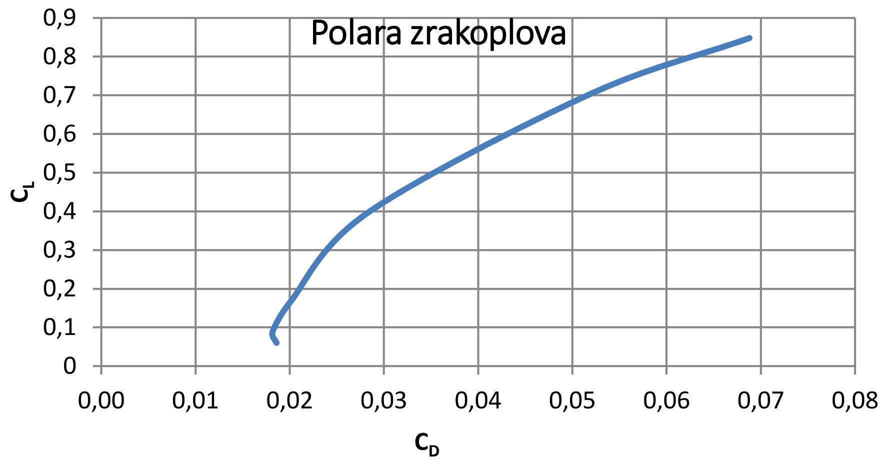
$V = \sqrt{\frac{2mg}{C_L \rho A}}$ ... brzina za horizontalni let

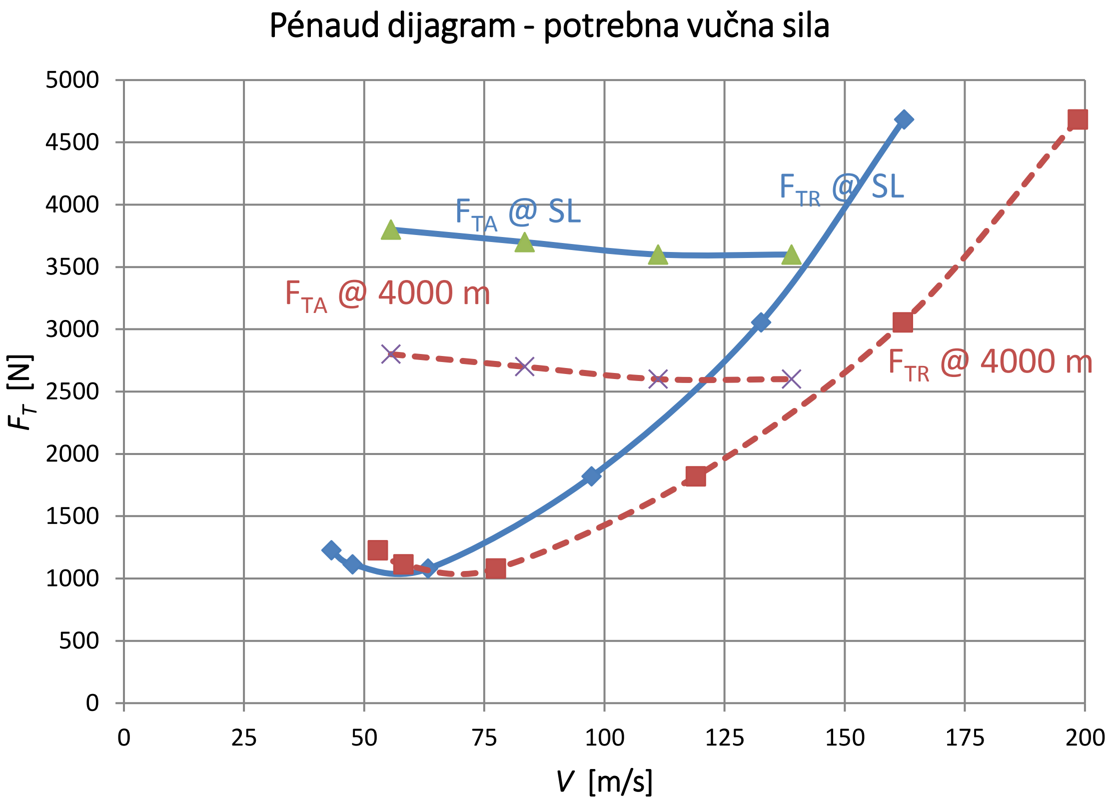
$P_R = F_{TR} \cdot V$

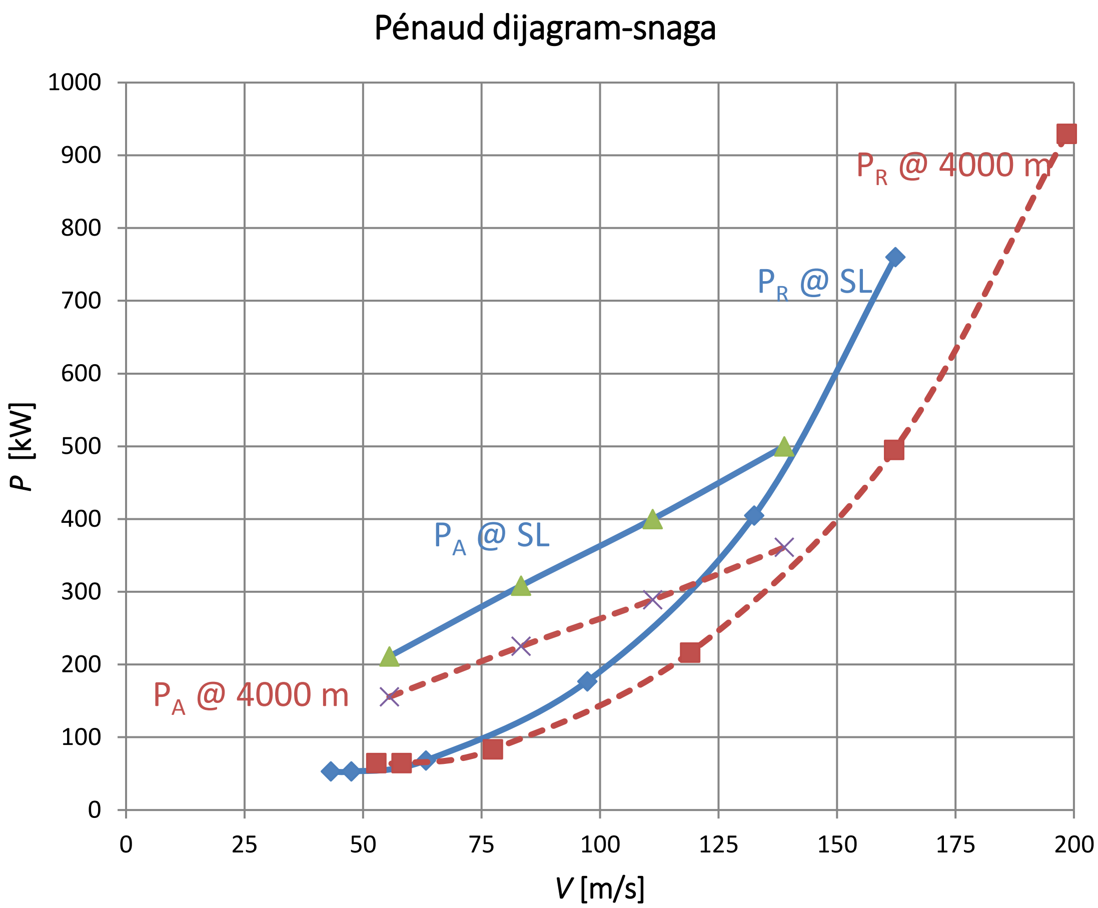
H [m]	*ρ* [kg/m^3]
0	1.225
4000	0.8194

C_L	C_D	F_{TR} [N]	*V* [m/s]@ 0 m	*V* [m/s]@ 4000 m	P_R [kW]@ 0 m	P_R [kW]@ 4000 m
0,06	0,0186	4683	162	198,5	760	929,5
0,09	0,0182	3055	132,5	162	405	495
0,167	0,0201	1818	97	119	177	216
0,395	0,0282	1078,5	63	77	68	83,4
0,7	0,0516	1114	47,5	58	53	65
0,848	0,0688	1226	43	53	53	64,7

V [km/h]	*V* [m/s]	F_{TA} [N]@ SL	F_{TA} [N]@ 4000 m	P_A [kW] @ SL	P_A [kW]@ 4000 m
200	55,6	3800	2800	211	155,6
300	83,3	3700	2700	308	225
400	111	3600	2600	400	289
500	139	3600	2600	500	361


Polara zrakoplova
c_L
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
0,00
0,01
0,02
0,03
0,04
0,05
0,06
0,07
0,08
c_D


Pénaud dijagram - potrebna vučna sila
F_T [N]
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
F_{TA} @ SL
F_{TR} @ SL
F_{TA} @ 4000 m
F_{TR} @ 4000 m
0
25
50
75
100
125
150
175
200
V [m/s]

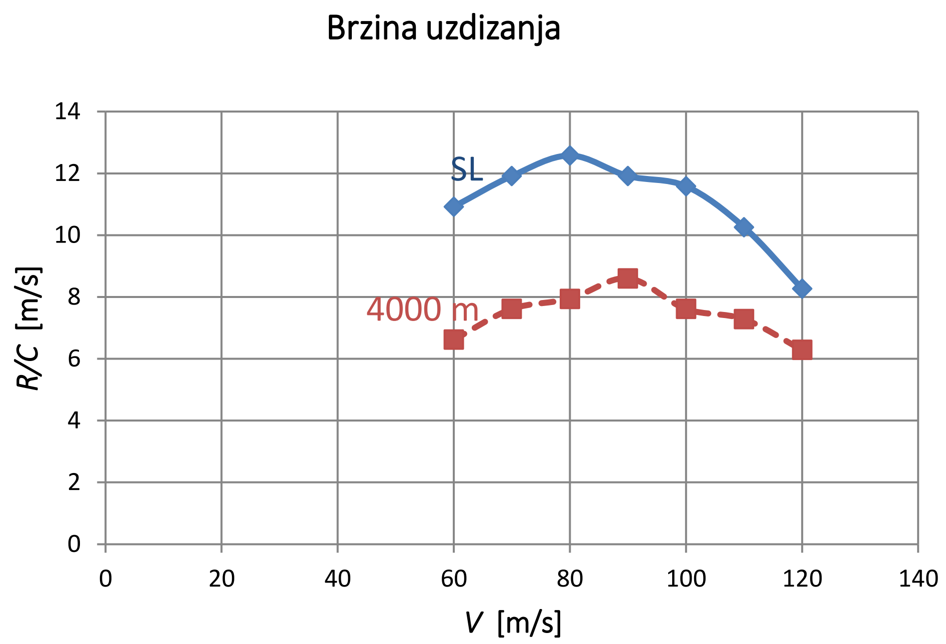


Za proizvoljne brzine iz Pénaudovog dijagrama snage očitavamo vrijednosti za raspoloživu i potrebnu snagu na određenoj visini i zatim prema jednadžbi za brzinu uzdizanja:

$$R/C = \frac{\Delta P}{F_G} = \frac{P_A - P_R}{F_G}$$

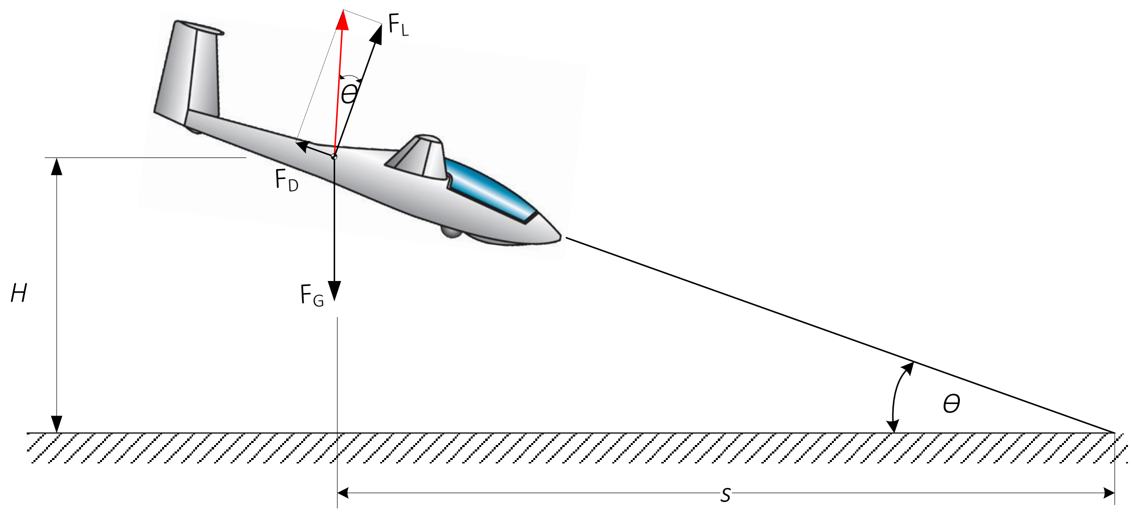
izračunavamo za svaku brzinu leta brzinu *R/C* i unosimo u tablicu.

V [m/s]	P_A [kW] @ SL	P_R [kW]@SL	Δ*P*@SL	P_A@4000	P_R@4000	Δ*P*@4000 m	*R/C*@SL	*R/C*@4000 m
60	225	60	165	175	75	100	10,92	6,62
70	270	90	180	195	80	115	11,91	7,61
80	300	110	190	210	90	120	12,58	7,94
90	330	150	180	240	110	130	11,91	8,61
100	365	190	175	260	145	115	11,58	7,61
110	400	245	155	290	180	110	10,26	7,28
120	430	305	125	305	210	95	8,27	6,29



5.7. Odredi minimalni kut planiranja i dolet jedrilice s visine 800 m, ako je polara jedrilice:

$$\boldsymbol{C_D = 0.018 + 0.04{C_L}^2}$$



Rješenje:

$$\tan\theta = \frac{F_D}{F_L} = \frac{C_D}{C_L} \quad \Rightarrow \quad \theta_{min} = \text{atan}\left(\frac{C_D}{C_L}\right)_{min}$$

$$\frac{C_D}{C_L} = \frac{0.018 + 0.04{C_L}^2}{C_L}$$

$$\frac{d}{dC_L}\left(\frac{C_D}{C_L}\right)_{min} = \frac{0.08C_L C_L - \left(0.018 + 0.04{C_L}^2\right)\cdot 1}{{C_L}^2} = \frac{0.04{C_L}^2 - 0.018}{{C_L}^2} = 0$$

$0.04{C_L}^2 - 0.018 = 0$

${C_L}^2 = \frac{0.018}{0.04} = 0.45 \qquad \rightarrow \quad C_L = 0.671$

$C_D = 0.032 + 0.055 \cdot 0.671^2 = 0.036$

$\theta_{min} = \text{atan}\frac{0.036}{0.671} = 3.07° = 3°\,4'$

$\tan\,\theta = \frac{1}{f} \quad \Longrightarrow \quad f = \frac{1}{\tan\theta} = \frac{1}{\tan 3.07} = 18.63$

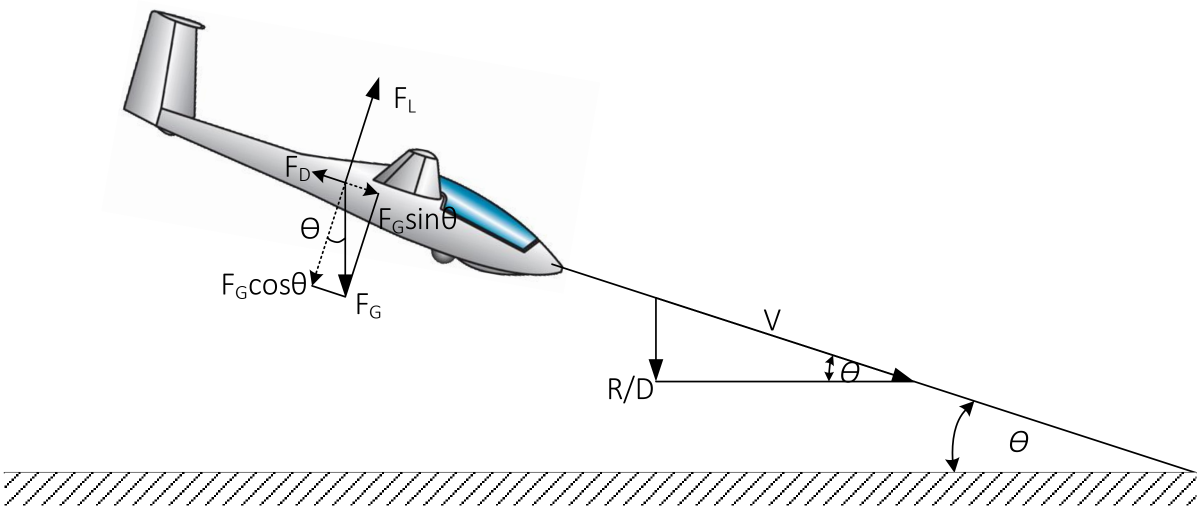
Odnos planiranja : 1:18.6

Dolet: $s = \frac{H}{\tan\theta} = H \cdot f = 800 \cdot 18.63 = 14\,907\ \text{m}$

5.8. Jedrilica ima površinu krila 41.26 m^2, a polara je $\boldsymbol{C_D = 0.02 + 0.04{C_L}^2}$. Masa prazne jedrilice je 975 kg, a masa dodatnog tereta 425 kg. Odredi za praznu i opterećenu jedrilicu u ISA/SL uvjetima:

a) minimalnu brzinu propadanja i

b) brzinu propadanja za najveći dolet.



Rješenje:

$R/D = V\sin\theta$

$$F_G \sin\theta = F_D$$

$$F_G \cos\theta = F_L$$

a) $F_D{}^2 + F_L{}^2 = F_G{}^2$

$$\left(\frac{1}{2}\rho V^2 A\right)^2 \left(C_D{}^2 + C_L{}^2\right) = F_G{}^2$$

$$\frac{1}{2}\rho V^2 A = \frac{F_G}{\sqrt{C_D{}^2 + C_L{}^2}}$$

$$\boxed{\boldsymbol{V = \sqrt{\frac{2F_G}{\rho A}} \cdot \frac{1}{\left(C_D{}^2 + C_L{}^2\right)^{1/4}}}}$$

$$\sin\theta = \frac{F_D}{F_G} = \frac{F_D}{\sqrt{F_D{}^2 + F_L{}^2}}$$

$$\boxed{\sin\theta = \frac{C_D \frac{1}{2}\rho V^2 A}{\sqrt{\left(\frac{1}{2}\rho V^2 A\right)^2 \left(C_D{}^2 + C_L{}^2\right)}} = \frac{C_D}{\left(C_D{}^2 + C_L{}^2\right)^{1/2}}}$$

$$R/D = V \cdot \sin\theta = \sqrt{\frac{2F_G}{\rho A}} \cdot \frac{1}{\left(C_D{}^2 + C_L{}^2\right)^{1/4}} \cdot \frac{C_D}{\left(C_D{}^2 + C_L{}^2\right)^{1/2}}$$

$$\boxed{R/D = \frac{C_D}{\left(C_D{}^2 + C_L{}^2\right)^{3/4}} \sqrt{\frac{2F_G}{\rho A}}}$$

$$C_L{}^2 \gg C_D{}^2 \quad \Rightarrow \quad C_D{}^2 + C_L{}^2 \approx C_L{}^2$$

$$\boxed{R/D = \frac{C_D}{C_L{}^{3/2}} \sqrt{\frac{2F_G}{\rho A}} \quad \Rightarrow \quad (R/D)_{min} = \left(\frac{C_D}{C_L{}^{3/2}}\right)_{min} \sqrt{\frac{2F_G}{\rho A}}}$$

$$\left(\frac{C_D}{C_L{}^{3/2}}\right)_{min} \quad \rightarrow \quad \frac{d}{dC_L}\left(\frac{C_D}{C_L{}^{3/2}}\right)_{min} = \frac{d}{dC_L}\left(\frac{0.02 + 0.04 C_L{}^2}{C_L{}^{3/2}}\right) = 0$$

$$\frac{0.08C_L{C_L}^{3/2}-\frac{3}{2}{C_L}^{1/2}\left(0.02+0.04{C_L}^2\right)}{{C_L}^3}=0$$

$$0.08{C_L}^{5/2}-0.03{C_L}^{1/2}-0.06{C_L}^{5/2}=0 \qquad /\div {C_L}^{1/2}$$

$$0.02{C_L}^2=0.03$$

$$C_L=\sqrt{\frac{0.03}{0.02}}=1.225$$

$$C_D=0.02+0.04\cdot 1.225^2=0.08$$

$$(R/D)_{min}=\left(\frac{C_D}{{C_L}^{3/2}}\right)_{min}\sqrt{\frac{2F_G}{\rho A}}=\frac{0.08}{1.225^{3/2}}\sqrt{\frac{2F_G}{1.225\cdot 41.26}}=0.011737\sqrt{F_G}$$

$m=975\ \text{kg} \qquad \rightarrow \quad (R/D)_{min}=0.011737\sqrt{975\cdot 9.81}=1.15\ \text{m/s}$

$m=975+425=1400\ \text{kg} \qquad \rightarrow \quad (R/D)_{min}=0.011737\sqrt{1400\cdot 9.81}=1.38\ \text{m/s}$

b)

$$\theta_{min}=\tan^{-1}\left(\frac{C_D}{C_L}\right)_{min}=\tan^{-1}\frac{1}{(C_L/C_D)_{max}}$$

$$\left(\frac{C_L}{C_D}\right)_{max} \Rightarrow \quad \frac{d}{dC_L}\left(\frac{C_L}{0.02+0.04{C_L}^2}\right)=0$$

$$\frac{0.02+0.04{C_L}^2-C_L\cdot 0.08C_L}{\left(0.02+0.04{C_L}^2\right)^2}=0$$

$$0.02-0.04{C_L}^2=0$$

$$C_L=0.707$$

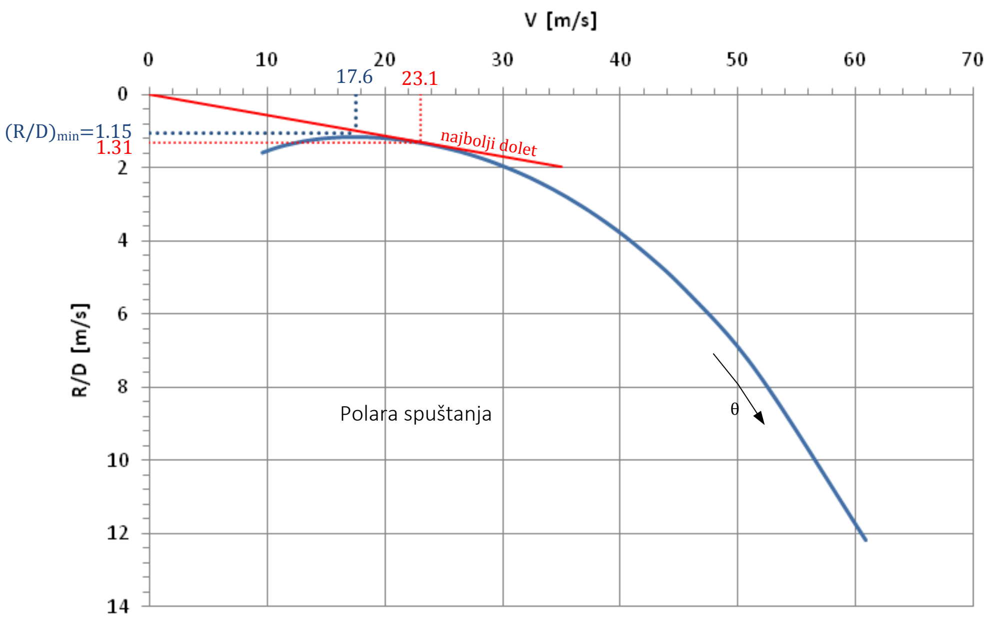
$$C_D=0.02+0.04\cdot 0.707^2=0.04$$

$$R/D=\frac{C_D}{{C_L}^{3/2}}\sqrt{\frac{2F_G}{\rho A}}=\frac{0.04}{0.707^{3/2}}\sqrt{\frac{2F_G}{1.225\cdot 41.26}}=0.01338\sqrt{F_G}$$

$m = 975\ \text{kg} \qquad \rightarrow\ R/D = 0.01338\sqrt{975 \cdot 9.81} = 1.31\ \text{m/s}$

$m = 975 + 425 = 1400\ \text{kg} \qquad \rightarrow\ R/D = 0.01338\sqrt{1400 \cdot 9.81} = 1.57\ \text{m/s}$

C_L	C_D	V [m/s]	$\sin\vartheta$	R/D [m/s]
0,10	0,02	60,9	0,20	12,17
0,20	0,02	43,4	0,11	4,66
0,30	0,02	35,5	0,08	2,78
0,40	0,03	30,7	0,07	2,02
0,50	0,03	27,5	0,06	1,65
0,60	0,03	25,1	0,06	1,44
0,70	0,04	23,2	0,06	1,31
0,80	0,05	21,7	0,06	1,24
0,90	0,05	20,5	0,06	1,19
1,00	0,06	19,4	0,06	1,16
1,10	0,07	18,5	0,06	1,15
1,20	0,08	17,7	0,06	1,14
1,30	0,09	17,0	0,07	1,15
1,40	0,10	16,4	0,07	1,15
1,50	0,11	15,9	0,07	1,16



6. DOLET I ISTRAJNOST

6.1. Izračunaj maksimalni dolet i istrajnost za zrakoplov Cessna Skylane. Specifična potrošnja goriva iznosi 0.27 kg/kWh. Korisnost propelera 0.8. Masa zrakoplova u polijetanju je 1338.1 kg, a može nositi 246 L goriva čija je gustoća 680 kg/m^3.

Veličina	b	A	m	C_{D0}	e	$\boldsymbol{AR}$
Vrijednost	10.9	16.2	1338	0.025	0.8	$\mathbf{7.3}$

Rješenje:

Bréguetova jednadžba za dolet zrakoplova s elisnim pogonom:

$$s = \frac{\eta}{g c_f} \left(\frac{C_L}{C_D}\right)_{max} \ln \frac{m_0}{m_1}$$

Ukupna količina goriva m_g:

$$m_g = \rho_g \cdot V_g = 680 \cdot 246 \cdot 10^{-3} = 167 \ \text{kg}$$

$$m_1 = m_0 - m_g = 1338 - 167 = 1171 \ \text{kg}$$

$$\left(\frac{C_L}{C_D}\right)_{max} = \frac{0.677}{0.05}$$ …poznato iz ranijih zadataka

$$s = \frac{0.8}{9.81 \cdot 0.27 \cdot \frac{1}{1000 \cdot 3600}} \frac{0.677}{0.05} \ln \frac{1338}{1171} = 1963 \ \text{km}$$

$$t = \frac{\eta}{c_f g^{3/2}} \left(\frac{{C_L}^{3/2}}{C_D}\right)_{max} \sqrt{2\rho A} \left(\frac{1}{\sqrt{m_1}} - \frac{1}{\sqrt{m_0}}\right)$$

$$\left(\frac{{C_L}^{3/2}}{C_D}\right)_{max} = 12.81$$ …poznato iz ranijih zadataka (minimalna potrebna snaga)

$$t = \frac{0.8}{0.27 \cdot \frac{1}{1000 \cdot 3600} \cdot 9.81^{3/2}} \cdot 12.81 \sqrt{2 \cdot 1.225 \cdot 16.2} \left(\frac{1}{\sqrt{1171}} - \frac{1}{\sqrt{1338}}\right) = 14.7 \ \text{h}$$

6.2. Odredi maksimalni dolet i istrajnost za zrakoplov Cessna Citation 3 za let na visini 6 700 m.

Podatci:

- ukupna masa $m_0 = 8988$ kg
- specifična potrošnja goriva $c_t = 1.667 \cdot 10^{-4} \quad 1/\mathrm{s}$
- količina goriva $V_g = 4236\ \ell \qquad (\rho_g = 800\ \mathrm{kg/m^3})$

Veličina	*b*	*A*	*m*	C_{D0}	*e*	*AR*
Vrijednost	16.2	29.5	8987.9	0.02	0.81	8.9

Rješenje:

a) $$s = \frac{2}{c_t}\sqrt{\frac{2g}{\rho A}}\left(\frac{\sqrt{C_L}}{C_D}\right)_{max}\left(\sqrt{m_0} - \sqrt{m_1}\right)$$

$$m_g = \rho_g \cdot V_g = 800 \cdot 4236 \cdot 10^{-3} = 3389\ \mathrm{kg}$$

$$m_1 = m_0 - m_g = 8988 - 3389 = 5599\ \mathrm{kg}$$

$$\left(\frac{\sqrt{C_L}}{C_D}\right)_{max} \rightarrow \begin{array}{l} C_{D0} = 3KC_L^{\,2} \quad \Rightarrow C_L = \sqrt{\dfrac{C_{D0}}{3K}} = 0.389 \\ C_D = \dfrac{4C_{D0}}{3} = 0.027 \end{array}$$

$$H = 6\,700\ \mathrm{m} \ \rightarrow \ \rho_{6700} = 0.6104\ \mathrm{kg/m^3}$$

$$s = \frac{2}{1.667 \cdot 10^{-4}}\sqrt{\frac{2 \cdot 9.81}{0.6104 \cdot 29.5}}\frac{\sqrt{0.389}}{0.027}\left(\sqrt{8988} - \sqrt{5599}\right) = 5\,779\ \mathrm{km}$$

b) $$t = \frac{1}{c_t}\left(\frac{C_L}{C_D}\right)_{max}\ln\frac{m_0}{m_1}$$

$$\left(\frac{C_L}{C_D}\right)_{max} \rightarrow \begin{array}{l} C_{D0} = KC_L^{\,2} \quad \Rightarrow C_L = 0.673 \\ C_D = 2C_{D0} = 0.04 \end{array}$$

$$t = \frac{1}{1.667 \cdot 10^{-4} \cdot 3600}\left(\frac{0.673}{0.04}\right)\ln\frac{8988}{5599} = 13.3\ \mathrm{h}$$

6.3. Dvomotorni mlazni zrakoplov čije su karakteristike: AR = 6.5, e = 0.87, A = 47 m^2, C_{D0} = 0.032 i c_t = 1 h^{-1}, leti na visini od 8 km, a ukupna masa na početku leta iznosila je 13 700 kg. Ako je poznata maksimalna dužina trajanja leta t = 5.5 h, potrebno je izračunati količinu goriva koju zrakoplov može ponijeti, te njegov maksimalni dolet koji može ostvariti s tom količinom goriva.

Rješenje:

$\boldsymbol{m_g} = ?$

$$t = \frac{1}{c_t}\left(\frac{C_L}{C_D}\right)_{max} \ln\frac{m_0}{m_1}$$

$$\left(\frac{C_L}{C_D}\right)_{max} \rightarrow \quad C_{D0} = KC_L^{\,2} \quad \Rightarrow C_L = \sqrt{\frac{C_{D0}}{K}} = \sqrt{\pi eARC_{D0}} = \sqrt{\pi \cdot 0.87 \cdot 6.5 \cdot 0.032} = 0.754$$

$$C_D = 2C_{D0} = 2 \cdot 0.032 = 0.064$$

$$\left(\frac{C_L}{C_D}\right)_{max} = \frac{0.754}{0.064} = 11.78$$

$$m_1 = m_0 - m_g \quad \rightarrow \; m_g = m_0 - m_1$$

$$\ln\frac{m_0}{m_1} = \frac{t \cdot c_t}{\left(\frac{C_L}{C_D}\right)_{max}}$$

$$m_0 = m_1 \cdot e^{\frac{t \cdot c_t}{(C_L/C_D)_{max}}} \quad \rightarrow \quad m_1 = m_0 \cdot e^{\frac{-t \cdot c_t}{(C_L/C_D)_{max}}}$$

$$m_1 = m_0 \cdot e^{\frac{-t \cdot c_t}{(C_L/C_D)_{max}}} = 13\,700 \cdot e^{\frac{-5.5 \cdot 1}{11.78}} = 8\,590 \;\; \text{kg}$$

$$m_g = m_0 - m_1 = 13\,700 - 8590 = 5\,110 \;\; \text{kg}$$

$\boldsymbol{s} = ?$

$$s = \frac{2}{c_t}\sqrt{\frac{2g}{\rho A}}\left(\frac{\sqrt{C_L}}{C_D}\right)_{max}\left(\sqrt{m_0} - \sqrt{m_1}\right)$$

$H = 8\,000 \text{ m} \;\rightarrow\; \rho_{8000} = 0.52578 \text{ kg/m}^3$

$$\left(\frac{\sqrt{C_L}}{C_D}\right)_{max} \rightarrow \quad \begin{aligned} &C_{D0} = 3KC_L^{\,2} \quad \Rightarrow C_L = \sqrt{\frac{C_{D0}}{3K}} = \sqrt{\frac{\pi \cdot 0.87 \cdot 6.5 \cdot 0.032}{3}} = 0.435 \\ &C_D = \frac{4C_{D0}}{3} = \frac{4 \cdot 0.032}{3} = 0.0427 \end{aligned}$$

$$\left(\frac{\sqrt{C_L}}{C_D}\right)_{max} = \frac{\sqrt{0.435}}{0.0427} = 15.45$$

$$s = \frac{2}{1} \cdot 3\,600 \cdot \sqrt{\frac{2 \cdot 9.81}{0.52578 \cdot 47}} \cdot 15.45 \cdot \left(\sqrt{13700} - \sqrt{8590}\right) = 2\,398 \text{ km}$$

6.4. Laki putnički zrakoplov poznate polare otpora $\boldsymbol{C_D = 0.019 + 0.047C_L^{\,2}}$ je u režimu maksimalnog doleta (*Maximum Range*) preletio 2260 km. Ako mu je masa na početku leta iznosila 8560 kg, odredi:

a) koliko je goriva potrošio te

b) vrijeme koje je proveo u zraku u tom režimu ako je letio na *H* = 3000 m

Zadano: *A* = 87 m², *b* = 25.9 m, *e* = 0.88, iskoristivost elise je 0.79, a specifična potrošnja goriva c_f = 3.2 N/kWh.

Rješenje:

$s = 2\,260 \text{ km}$

$$s = \frac{\eta}{gc_f}\left(\frac{C_L}{C_D}\right)_{max} \ln\frac{m_0}{m_1}$$

$$m_g = m_0 - m_1$$

$$AR = \frac{b^2}{A} = \frac{25.9^2}{87} = 7.7$$

$$\left(\frac{C_L}{C_D}\right)_{max} \rightarrow \quad C_{D0} = K{C_L}^2 \quad \Rightarrow C_L = \sqrt{\frac{C_{D0}}{K}} = \sqrt{\pi eARC_{D0}} = \sqrt{\pi \cdot 0.88 \cdot 7.7 \cdot 0.019} = 0.636$$

$$C_D = 2C_{D0} = 2 \cdot 0.019 = 0.038$$

$$\left(\frac{C_L}{C_D}\right)_{max} = \frac{0.636}{0.038} = 16.73$$

$$ln\frac{m_0}{m_1} = \frac{s \cdot g \cdot c_f}{\eta\left(\frac{C_L}{C_D}\right)_{max}} \quad \Longrightarrow \quad m_0 = m_1 \cdot e^{\frac{s \cdot g \cdot c_f}{\eta (C_L/C_D)_{max}}}$$

$$m_1 = m_0 \cdot e^{\frac{-s \cdot g \cdot c_f}{\eta (C_L/C_D)_{max}}} \qquad = 8560 \cdot e^{\frac{-2260 \cdot 1000 \cdot 9.81 \cdot \frac{3.2}{9.81.1000 \cdot 3600}}{0.79 \cdot 16.73}} = 7\,353 \ \text{kg}$$

$$m_g = m_0 - m_1 = 8560 - 7353 = 1207 \ \text{kg}$$

b) $H = 3\,000 \ \text{m} \ \rightarrow \ \rho_{3000} = 0.9093 \ \text{kg/m}^3$

$$t = \frac{\eta}{c_f g^{3/2}} \frac{{C_L}^{3/2}}{C_D} \sqrt{2\rho A}\left(\frac{1}{\sqrt{m_1}} - \frac{1}{\sqrt{m_0}}\right)$$

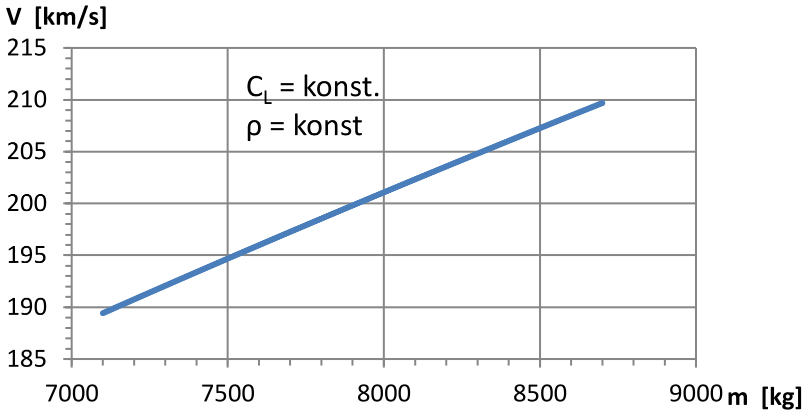
$$t = \frac{0.79}{\frac{3.21}{1000 \cdot 9.81} \cdot 9.81^{3/2}} \cdot \frac{0.636^{3/2}}{0.038} \sqrt{2 \cdot 0.9093 \cdot 87}\left(\frac{1}{\sqrt{7353}} - \frac{1}{\sqrt{8560}}\right) = 11.3 \ \text{h}$$

$$F_L = F_G \qquad \text{odnosno} \qquad \frac{1}{2}\rho V^2 C_L A = F_G$$

$$\left.\begin{aligned} \frac{1}{2}\rho V_0^2 C_L A &= m_0 g \\ \frac{1}{2}\rho V_1^2 C_L A &= m_1 g \end{aligned}\right\} \qquad C_L = 0.636 = \text{konst. u režimu za maksimalni dolet}$$

$$V_0 = \sqrt{\frac{2m_0 g}{\rho C_L A}} = \sqrt{\frac{2 \cdot 8560 \cdot 9.81}{0.9093 \cdot 0.636 \cdot 87}} = 57.8 \ \text{m/s} \quad \approx 208 \, \text{km/h}$$

$$V_1 = \sqrt{\frac{2m_1 g}{\rho C_L A}} = \sqrt{\frac{2 \cdot 7353 \cdot 9.81}{0.9093 \cdot 0.636 \cdot 87}} = 53.6 \ \text{m/s} \quad \approx 193 \, \text{km/h}$$



6.5. Odredi kako međusobno zavise dolet mlaznog aviona i masa plaćenog tereta (*payload*), koristeći Breguetovu formulu.

Zadano:

Max. dopuštena masa za uzlijetanje	MTOM = 60 000 kg
Masa aviona i posade, bez goriva	DOM = 40 000 kg
Max. količina goriva u spremnicima	m_{Fmax} = 12 000 kg
Max. dopuštena masa plaćenog tereta	m_{Pmax} = 16 000 kg
Polara aviona	$\boldsymbol{C_D = 0.024 + 0.049 {C_L}^2}$
Površina krila	A = 99 m^2
Specifična potrošnja TSFC	$\boldsymbol{c_t = 0.4\, h^{-1}}$
Visina leta	H = 8 000 m

Rješenje:

$$c_t = 0.4\ \mathrm{h}^{-1} = \frac{0.4}{3600}\ s^{-1} = 0.000111\ \mathrm{s}^{-1}$$

$$R_{max} \rightarrow \left(\frac{\sqrt{C_L}}{C_D}\right)_{max}$$

$$\left(\frac{\sqrt{C_L}}{C_D}\right)_{max} \rightarrow \begin{array}{l} C_{D0} = 3K{C_L}^2 \quad \Rightarrow C_L = \sqrt{\dfrac{C_{D0}}{3K}} = \sqrt{\dfrac{0.024}{3 \cdot 0.049}} = 0.404 \\ C_D = \dfrac{4C_{D0}}{3} = \dfrac{4 \cdot 0.024}{3} = 0.032 \end{array}$$

$$\left(\frac{\sqrt{C_L}}{C_D}\right)_{max} = \frac{\sqrt{0.404}}{0.032} = 19.86$$

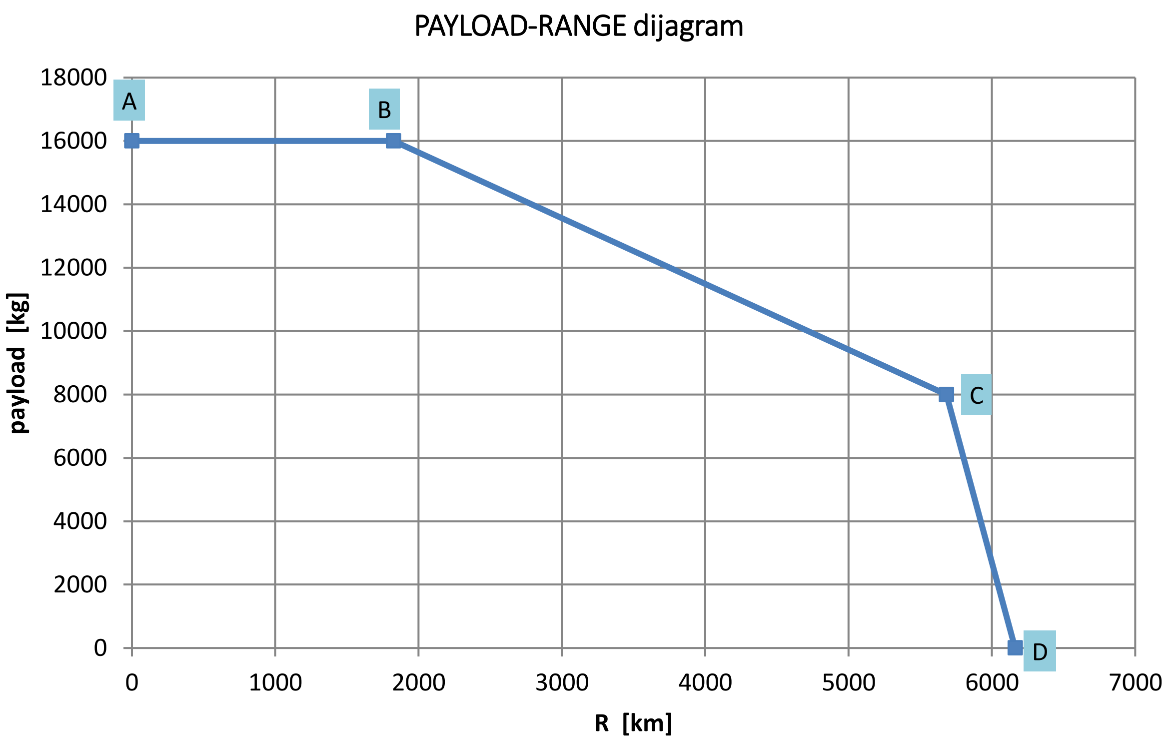
$$H = 8\,000\ \mathrm{m} \ \rightarrow \ \rho_{8000} = 0.5258\ \ \mathrm{kg/m^3}$$

Dolet – mlazni pogon:

$$R = \frac{1}{c\ t}\sqrt{\frac{8g}{\rho A}}\left(\frac{\sqrt{C_L}}{C_D}\right)_{max} \left(\sqrt{m_0} - \sqrt{m_1}\right) \qquad (1)$$

$$m_g = m_0 - m_1$$

		m_0 [kg]	m_F [kg]	m_P [kg]	m_1 [kg]	R [km]
						(1)
A	DOM + m_{Pmax}; $m_F = 0$	56000	0	16000	56000	0
B	DOM + m_{Pmax}; MTOM	60000	4000	16000	56000	1826
C	DOM + m_{Fmax}; MTOM	60000	12000	8000	48000	5684
D	DOM + m_{Fmax}; $m_P = 0$	52000	12000	0	40000	6162



7. UZLIJETANJE

7.1. Odrediti duljinu zaleta za Cessnu Citatation u ISA/SL uvjetima. USS je betonirana s koeficijentom trenja $\boldsymbol{\mu = 0.02}$. Tijekom polijetanja, koeficijent uzgona je ograničen na $\boldsymbol{C_{L_{max}} = 1}$. Dok je zrakoplov na zemlji, udaljenost krila od zemlje iznosi $\boldsymbol{h = 1.83}$ **m**. Brzina odvajanja treba biti 20% veća od V_{min}.

Rješenje:

$$s_{LO} = \frac{1.44 \cdot F_G^2}{g\rho A C_{L_{max}}\{F_T - [F_D + \mu(F_G - F_L)]_{sr}\}}$$

$$F_G = mg = 8987.9 \cdot 9.81 = 88\,171.3 \text{ N}$$

$$F_T = 2 \cdot 16\,236 = 32\,472 \text{ N}$$

$$\phi = \frac{\left(16 \cdot \frac{h}{b}\right)^2}{1 + \left(16 \cdot \frac{h}{b}\right)^2} = \frac{\left(16 \cdot \frac{1.83}{16.2}\right)^2}{1 + \left(16 \cdot \frac{1.83}{16.2}\right)^2} = 0.7656$$

$$V_{LO} = 1.2 V_{stall} = 1.2\sqrt{\frac{2mg}{\rho A C_{L_{max}}}} = 1.2\sqrt{\frac{2 \cdot 88171.3}{1.225 \cdot 29.5 \cdot 1}} = 83.83 \text{ m/s}$$

$$V_{sr} = 0.7 \cdot V_{LO} = 0.7 \cdot 83.83 = 58.7 \text{ m/s}$$

$$F_D = \left(C_{D0} + \phi\frac{C_{Lmax}^2}{\pi e AR}\right)\frac{1}{2}\rho V_{sr}^2 A = \left(0.02 + \frac{0.7656 \cdot 1^2}{\pi \cdot 0.81 \cdot 8.9}\right)\frac{1}{2} \cdot 1.225 \cdot 58.7^2 \cdot 29.5 = 3350 \text{ N}$$

$$F_L = C_L \frac{1}{2}\rho V_{sr}^2 A = 1 \cdot \frac{1}{2} \cdot 1.225 \cdot 58.7^2 \cdot 29.5 = 62\,259 \text{ N}$$

$$s_{LO} = \frac{1.44 \cdot 88171.3^2}{9.81 \cdot 1.225 \cdot 29.5 \cdot 1\{32472 - [3350 + 0.02(88171.3 - 62259)]_{sr}\}} = 1104 \text{ m}$$

7.2. Odredi duljinu zaleta za zrakoplov 747-100. Podaci:

- Težina zrakoplova $\boldsymbol{F_G = 733\ 000\ \mathrm{lb} = 3\ 260\ 000\ \mathrm{N}}$
- Raspoloživi potisak za 1 motor $\boldsymbol{F_T = 46100 - 46.7 \cdot V + 0.0467 \cdot V^2}$ $[\mathbf{lb}]$, $\boldsymbol{V}\ [\mathbf{fps}]$
- 4 turbofan motora JT9D-7A
- Površina krila A = 5500 ft^2= 511 m^2
- Oswaldov koeficijent e = 0.7
- Raspon krila b = 196 ft = 59.74 m
- Omjer udaljenosti krila od zemlje i raspona krila $\boldsymbol{h/b = 0.08}$
- Koeficijent trenja podloge $\boldsymbol{\mu = 0.02}$
- Štetni otpori - otpor ravne ploče površine A_p = 100 ft^2 = 9.29 m^2 s koeficijentom otpora C_{Dp}= 1

  $$\boldsymbol{C_{D0}A = C_{Dp}A_p \quad \rightarrow \quad C_{D0} = C_{Dp}\frac{A_p}{A} = 1 \cdot \frac{9.29}{511} = 0.0182}$$
- Max. koeficijent uzgona C_{Lmax}= 1.8
- Brzina odvajanja V_{LO}= 1.1 V_{min}

Rješenje:

$$s_{LO} = \frac{1.21 \cdot F_G^2}{g\rho AC_{L_{max}}\{F_T - [F_D + \mu(F_G - F_L)]_{sr}\}}$$

$$V_{stall} = \sqrt{\frac{2F_G}{\rho AC_{L_{max}}}} = \sqrt{\frac{2 \cdot 3260000}{1.225 \cdot 511 \cdot 1.8}} = 76.1\ \mathrm{m/s}\ \ (V_{stall} = 274\ \ \mathrm{km/h})$$

$$V_{LO} = 1.1V_{stall} = 1.1 \cdot 76.1 = 83.7\quad \mathrm{m/s}\quad (V_{LO} = 301\ \ \mathrm{km/h})$$

$$V_{sr} = 0.7 \cdot V_{LO} = 0.7 \cdot 83.7\ = 59.2\quad \mathrm{m/s}\ \ (V_{sr} = 213\ \ \mathrm{km/h})$$

$$V_{sr} = 59.2\ \ \mathrm{m/s} = 194\ \mathrm{ft/s}$$ [1]

$$F_T = 46100 - 46.7 \cdot V_{sr} + 0.0467 \cdot V_{sr}^2 = 46100 - 46.7 \cdot 194 + 0.0467 \cdot 194^2 = 38\ 800\ \ \mathrm{lb}$$

[1] $1\ lb = 0.45359237\ \mathrm{kg}$
$1\ lb_f = 4.44822\ \mathrm{N}$
$1\ ft = 0.3048\ \mathrm{m}$

$$F_T = 38\,800 \text{ lb} = 172\,591 \text{ N}$$

$$F_{T_R} = 4F_T = 4 \cdot 172\,591 = 690\,364 \text{ N}$$

$$\phi = \frac{\left(16 \cdot \frac{h}{b}\right)^2}{1 + \left(16 \cdot \frac{h}{b}\right)^2} = \frac{(16 \cdot 0.08)^2}{1 + (16 \cdot 0.08)^2} = 0.621$$

$$AR = \frac{b^2}{A} = \frac{59.74^2}{511} = 7$$

$$F_D = C_D \frac{1}{2} \rho {V_{sr}}^2 A = \left(C_{D0} + \phi \frac{C_{L_{max}}^2}{\pi e AR}\right) \frac{1}{2} \rho {V_{sr}}^2 A =$$

$$= \left(0.0182 + 0.621 \frac{1.8^2}{\pi \cdot 0.7 \cdot 7}\right) \frac{1}{2} \cdot 1.225 \cdot 59.2^2 \cdot 511 = 163\,335 \text{ N}$$

$$F_L = C_L \frac{1}{2} \rho {V_{sr}}^2 A = 1.8 \cdot \frac{1}{2} \cdot 1.225 \cdot 59.2^2 \cdot 511 = 1\,974\,400 \text{ N}$$

$$s_{LO} = \frac{1.21 \cdot 3\,260\,000^2}{9.81 \cdot 1.225 \cdot 511 \cdot 1.8\{690\,364 - [163\,335 + 0.02(3\,260\,000 - 1\,974\,400)]_{sr}\}} =$$

$$= 2321 \text{ m}$$

7.3. Izračunajte potrebnu duljinu za zalet sa betonirane staze za dvomotorni mlazni zrakoplov Fairchild Republic A-10, pri masi 10504.3 kg i sa slijedećim karakteristikama:

- $S_w = A = 47 \text{ m}^2$
- AR = 6.5
- e = 0.87
- $C_{D0} = 0.032$
- $F_{T1} = 40\ 298$ N na razini mora
- Udaljenost krila od tla je 1.53 m
- $\boldsymbol{C_{L_{max}} = 0.8}$

Rješenje:

$$s_{LO} = \frac{1.44 \cdot F_G^2}{g\rho A C_{L_{max}}\{F_T - [F_D + \mu(F_G - F_L)]_{sr}\}}$$

$$F_G = mg = 10504.3 \cdot 9.81 = 103\ 047 \text{ N}$$

$$F_T = 2 \cdot 40\ 298 = 80\ 596 \text{ N}$$

$$AR = \frac{b^2}{A} \quad \Rightarrow \quad b = \sqrt{AR \cdot A} = \sqrt{6.5 \cdot 47} = 17.5 \text{ m}$$

$$\phi = \frac{\left(16 \cdot \frac{h}{b}\right)^2}{1 + \left(16 \cdot \frac{h}{b}\right)^2} = \frac{\left(16 \cdot \frac{1.53}{17.5}\right)^2}{1 + \left(16 \cdot \frac{1.53}{17.5}\right)^2} = 0.66$$

$$V_{LO} = 1.2 V_{stall} = 1.2\sqrt{\frac{2mg}{\rho A C_{L_{max}}}} = 1.2\sqrt{\frac{2 \cdot 10\ 504.3 \cdot 9.81}{1.225 \cdot 47 \cdot 0.8}} = 80.3 \text{ m/s}$$

$$V_{sr} = 0.7 \cdot V_{LO} = 0.7 \cdot 80.3 = 56.2 \text{ m/s}$$

$$F_D = \left(C_{D0} + \phi\frac{C_{L_{max}}^2}{\pi e AR}\right)\frac{1}{2}\rho V_{sr}^2 A = \left(0.032 + 0.66\frac{0.8^2}{\pi \cdot 0.87 \cdot 6.5}\right)\frac{1}{2} \cdot 1.225 \cdot 56.2^2 \cdot 4 = 5073 \text{ N}$$

$$F_L = C_L \frac{1}{2}\rho V_{sr}^2 A = 0.8 \cdot \frac{1}{2} \cdot 1.225 \cdot 56.2^2 \cdot 47 = 72\ 740 \text{ N}$$

$$s_{LO} = \frac{1.44 \cdot 103\ 047^2}{9.81 \cdot 1.225 \cdot 47 \cdot 0.8\{80\ 596 - [5\ 073 + 0.02(103\ 047 - 72\ 740)]_{sr}\}} = 452 \text{ m}$$

7.4. Odredite duljinu zaleta (rulanja po pisti) jednomotornog klipnog zrakoplova Beechcraft Bonanza u uvjetima ISA/SL sa betonirane staze ako ima slijedeće karakteristike:

A [m²]	*AR*	*e*	*m* [kg]	C_{D0}	P_{mot} [kW]	*η*	C_{Lmax}
16.84	6.2	0.91	1350	0.027	253.7	0.83	1.1

Udaljenost krila od tla je 1.22 m.

Rješenje:

$$s_{LO} = \frac{1.44 \cdot F_G^2}{g\rho A C_{L_{max}}\{F_T - [F_D + \mu(F_G - F_L)]_{sr}\}}$$

$$F_G = mg = 1350 \cdot 9.81 = 13\,243.5 \text{ N}$$

$$V_{LO} = 1.2V_{stall} = 1.2\sqrt{\frac{2mg}{\rho A C_{L_{max}}}} = 1.2\sqrt{\frac{2 \cdot 1350 \cdot 9.81}{1.225 \cdot 16.84 \cdot 1.1}} = 41 \text{ m/s}$$

$$V_{sr} = 0.7 \cdot V_{LO} = 0.7 \cdot 41 = 28.7 \text{ m/s}$$

$$F_T = \frac{P_A}{V_{sr}} = \frac{\eta \cdot P_{mot}}{V_{sr}} = \frac{0.83 \cdot 253700}{28.7} = 7\,337 \text{ N}$$

$$AR = \frac{b^2}{A} \quad \Longrightarrow \quad b = \sqrt{AR \cdot A} = \sqrt{6.2 \cdot 16.84} = 10.22 \text{ m}$$

$$\phi = \frac{\left(16 \cdot \frac{h}{b}\right)^2}{1 + \left(16 \cdot \frac{h}{b}\right)^2} = \frac{\left(16 \cdot \frac{1.22}{10.22}\right)^2}{1 + \left(16 \cdot \frac{1.22}{10.22}\right)^2} = 0.785$$

$$F_D = \left(C_{D0} + \phi\frac{C_{L_{max}}^2}{\pi e AR}\right)\frac{1}{2}\rho V_{sr}^2 A = \left(0.027 + 0.785\frac{1.1^2}{\pi \cdot 0.91 \cdot 6.2}\right)\frac{1}{2} \cdot 1.225 \cdot 28.7^2 \cdot 16.84$$
$$= 684.5 \text{ N}$$

$$F_L = C_L\frac{1}{2}\rho V_{sr}^2 A = 1.1 \cdot \frac{1}{2} \cdot 1.225 \cdot 28.7^2 \cdot 16.84 = 9\,345.4 \text{ N}$$

$$s_{LO} = \frac{1.44 \cdot 13\,243.5^2}{9.81 \cdot 1.225 \cdot 16.84 \cdot 1.1\{7\,337 - [684.5 + 0.02(13\,243.5 - 9\,345.4)]_{sr}\}} = 173 \text{ m}$$

7.5. Ako je raspoloživa duljina piste za zalijetanje “ground roll“ jednaka 1700 m. Izračunajte maksimalnu dopuštenu masu velikog putničkog zrakoplova (tipa Airbus 380) za polijetanje s obzirom na zahtjev da mu duljina zaleta bude maksimalno 1700 m. Zadani su podaci o zrakoplovu:

A [m^2]	b [m]	e	μ	C_{D0}	F_T [kN]	C_{Lmax}
511	60	0.91	0.03	0.0364	705.2	1.8

Udaljenost krila od tla je 5 m.

Rješenje:

$$s_{LO} = \frac{1.44 \cdot (m \cdot g)^2}{g\rho A C_{L_{max}}\{F_T - [F_D + \mu(F_G - F_L)]_{sr}\}}$$

$$V_{LO} = 1.2 V_{stall} = 1.2\sqrt{\frac{2mg}{\rho A C_{L_{max}}}} = 1.2\sqrt{\frac{2 \cdot m \cdot 9.81}{1.225 \cdot 511 \cdot 1.8}} = 0.158\sqrt{m} \qquad \mathrm{m/s}$$

$$V_{sr} = 0.7 \cdot V_{LO} = 0.7 \cdot 0.158\sqrt{m} = 0.111\sqrt{m} \qquad \mathrm{m/s}$$

$$AR = \frac{b^2}{A} = \frac{60^2}{511} = 7$$

$$\phi = \frac{\left(16 \cdot \frac{h}{b}\right)^2}{1 + \left(16 \cdot \frac{h}{b}\right)^2} = \frac{\left(16 \cdot \frac{5}{60}\right)^2}{1 + \left(16 \cdot \frac{5}{60}\right)^2} = 0.64$$

$$F_D = \left(C_{D0} + \phi\frac{C_{L_{max}}^2}{\pi e AR}\right)\frac{1}{2}\rho V_{sr}^2 A$$

$$F_D = \left(0.0364 + 0.64\frac{1.8^2}{\pi \cdot 0.91 \cdot 7}\right)\frac{1}{2} \cdot 1.225\left(0.111\sqrt{m}\right)^2 511 = 0.54 \cdot m \qquad \mathrm{N}$$

$$F_L = C_L\frac{1}{2}\rho V^2 A = 1.8 \cdot \frac{1}{2} \cdot 1.225 \cdot \left(0.111\sqrt{m}\right)^2 \cdot 511 = 6.93 \cdot m \qquad \mathrm{N}$$

$$s_{LO} = \frac{1.44 \cdot (m \cdot g)^2}{g\rho A C_{L_{max}}\{F_T - [F_D + \mu(F_G - F_L)]_{sr}\}}$$

$$1\,700 = \frac{1.44 \cdot 9.81^2 \cdot m^2}{9.81 \cdot 1.225 \cdot 511 \cdot 1.8\{705\,200 - [0.54 \cdot m + 0.03(9.81 \cdot m - 6.93 \cdot m)]_{sr}\}}$$

$$1\,700\{705\,200 - [0.54 \cdot m + 0.03(9.81 \cdot m - 6.93 \cdot m)]_{sr}\} = 0.0125 \cdot m^2$$

$$m^2 + 85\,190 \cdot m - 9.56 \cdot 10^{10} = 0$$

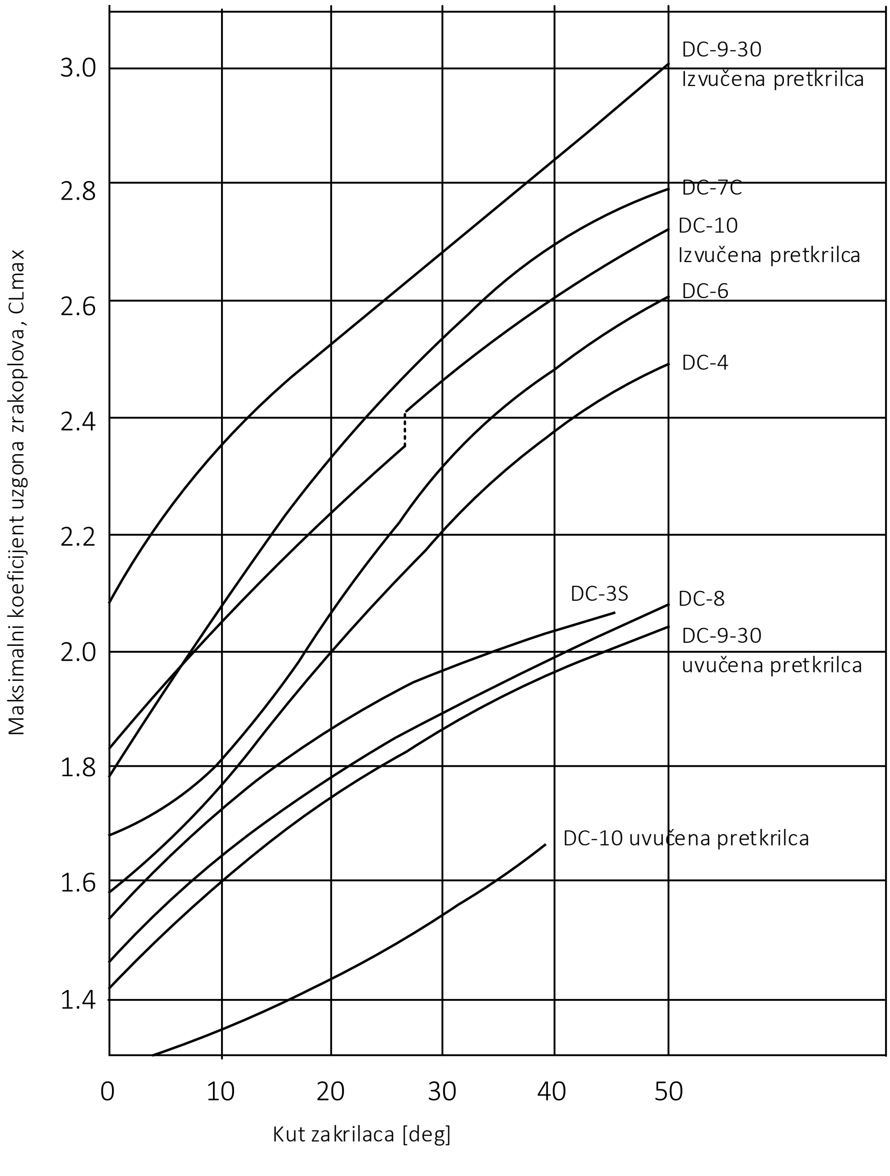
$$m_{1,2} = \frac{-85\,190 \pm \sqrt{85\,190^2 + 4 \cdot 9.56 \cdot 10^{10}}}{2} = \frac{-85\,190 \pm 624\,225}{2}$$

$$m = 269\,518\ \text{kg}$$

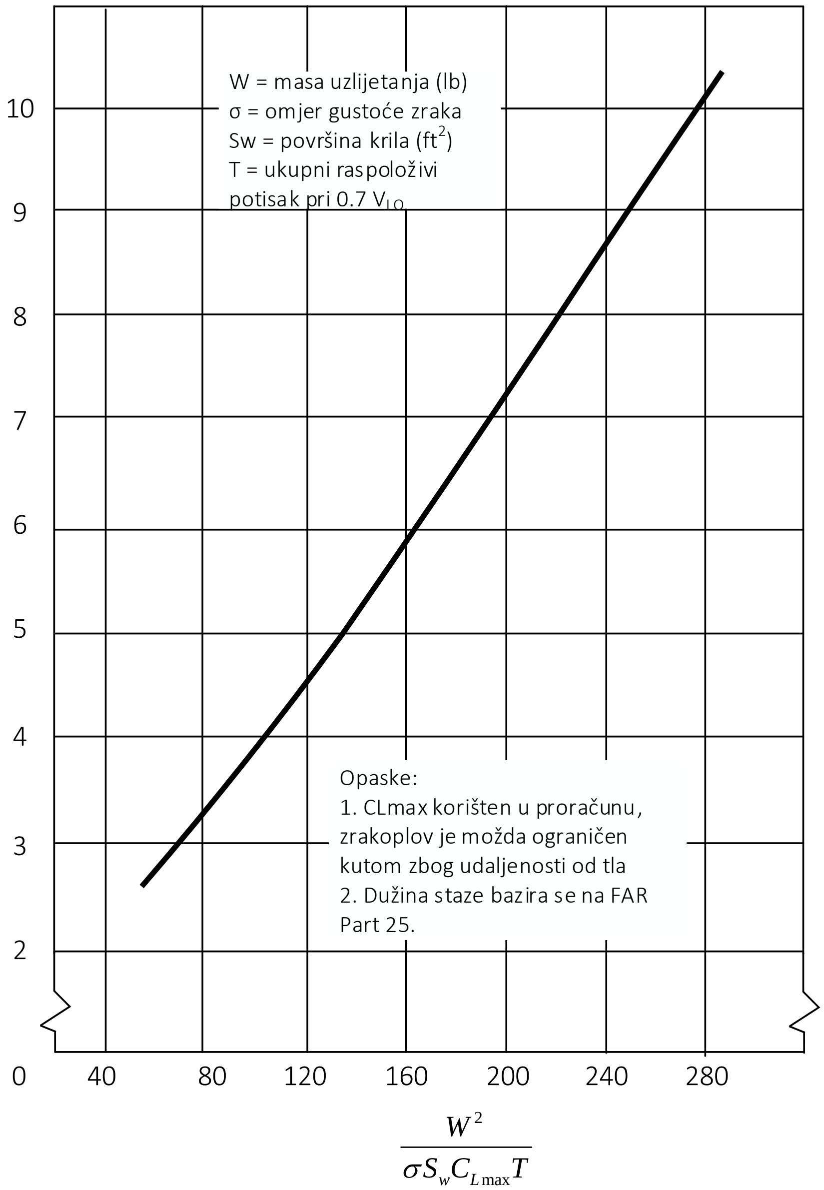
7.6. Prilikom uzlijetanja iz San Francisca, dvomotorni DC-9-30 ima masu 45 900 kg. Površina krila mu je 93 m^2. Aerodrom ima tlak zraka kao na razini mora. Temperatura je 27° C. Motori imaju statički potisak (V = 0) od 66 218 N svaki, ali se prilikom prosječne brzine uzlijetanja od 70% V_{LO} pojavljuju gubici od 14% u potisku zbog elise i otpora (*ram drag*) pri ubrzavanju. Zakrilca su na 15°, a pretkrilca su izvučena. Potrebno je odrediti:

a) FAR potrebnu duljinu uzlijetanja

b) Brzinu penjanja pri 1.2 V_{stall}



Dijagram 1. Maksimalni koeficijent uzgona zrakoplova DC



Dijagram 2. FAR duljina uzlijetanja

Rješenje:

$T = 27\,°\,C = 300.15\ \mathrm{K}$

$$p = \rho R T \quad \Longrightarrow \quad \rho = \frac{p}{RT} = \frac{101325}{287 \cdot 300.15} = 1.1768\ \ \mathrm{kg/m^3}$$

$$\sigma = \frac{\rho}{\rho_0} = \frac{1.1768}{1.225} = 0.96$$

Za kut zakrilaca od 15° iz dijagrama 1. očitan je najveći koeficijent uzgona:

$\delta_F = 15\,° \quad \Longrightarrow \quad C_{L_{max}} = 2.42$

$F_T = 66218 \cdot 0.86 \cdot 2 = 113\,895\ \text{N}$

$$\frac{(m \cdot g)^2}{\sigma A C_{L_{max}} F_T} \cdot 0.0211 = \frac{(45\,900 \cdot 9.81)^2}{0.96 \cdot 93 \cdot 2.42 \cdot 113\,895} \cdot 0.0211 = 174$$

$s_{LO} = 6.35 \cdot 305 = 1\,937\ \text{m}$ (iz dijagrama 2.)

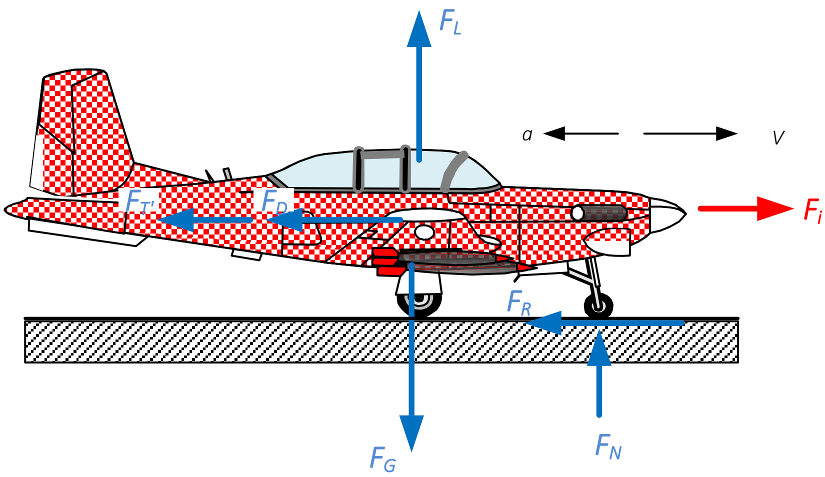
$$V_{LO} = 1.2 V_{stall} = 1.2 \sqrt{\frac{2mg}{\rho A C_{L_{max}}}} = 1.2 \sqrt{\frac{2 \cdot 45\,900 \cdot 9.81}{1.1768 \cdot 93 \cdot 2.42}} = 70\ \ \text{m/s}$$

8. SLIJETANJE

8.1. Odrediti potrebnu duljinu USS za zaustavljanje Cessne Citatation pri slijetanju. Nakon dodira sa zemljom koriste se spojleri koji reduciraju uzgon na 0, ne upotrebljava se revers potiska. Izvlačenje spojlera povećava parazitni otpor za 10%. Težina goriva se može zanemariti. Maksimalni koeficijent uzgona prije dodira sa zemljom iznosi 2.5 zbog potpuno izvučenih zakrilaca. Staza je betonirana s koeficijentom trenja pri kočenju $\boldsymbol{\mu = 0.4}$.

Zadano:

A [m^2]	F_G [N]	C_L	μ	C_{D0}	C'_{D0}	C_{Lmax}
29.5	54944	0	0.4	0.02	1.1 C_{D0}	2.5



Rješenje:

$$s_L = \frac{s^2 \cdot F_G^2}{g\rho A C_{Lmax}[F_D + \mu(F_G - F_L)]_{sr}}$$

$$V_{TD} = 1.3V_{stall} = 1.3\sqrt{\frac{2F_G}{\rho A C_{Lmax}}} = 1.2\sqrt{\frac{2 \cdot 54\,944}{1.225 \cdot 29.5 \cdot 2.5}} = 45.3 \ \ \mathrm{m/s}$$

$$V_{sr} = 0.7 \cdot V_T = 0.7 \cdot 45.3 \ = 31.7 \ \ \mathrm{m/s}$$

$$C'_{D0} = 1.1C_{D0} = 1.1 \cdot 0.02 = 0.022$$

$$F_D = \left(C'_{D0} + \phi \underbrace{\frac{C_L^2}{\pi e AR}}_{0} \right) \frac{1}{2} \rho V^2 A = C'_{D0} \frac{1}{2} \rho {V_{sr}}^2 A = 0.022 \cdot \frac{1}{2} \cdot 1.225 \cdot 31.7^2 \cdot 29.5 = 399.5 \text{ N}$$

$$C_L = 0 \rightarrow F_L = 0 \ \ N$$

$$s_L = \frac{1.3^2 \cdot 54\,944^2}{9.81 \cdot 1.225 \cdot 29.5 \cdot 2.5 [399.5 + 0.4(54\,944 - 0)]_{sr}} = 257 \text{ m}$$

8.2. Odredi potrebnu duljinu staze za zaustavljanje zrakoplova Boeing 747-100. Podaci:

- Težina zrakoplova $\boldsymbol{F_G = 2\,500\,000}$ **N**
- $\boldsymbol{F_T = 0}$ **N**
- A = 511 m^2
- Koeficijent trenja pri kočenju $\boldsymbol{\mu = 0.4}$
- Štetni otpori - otpor ravne ploče površine A_p = 9.29 m^2 s koeficijentom otpora C_{Dp}= 1

  $$\boldsymbol{C_{D0} A = C_{Dp} A_p \quad \rightarrow \quad C_{D0} = C_{Dp} \frac{A_p}{A} = 1 \cdot \frac{9.29}{511} = 0.0182}$$

- C_L = 0
- Brzina pri dodiru s pistom V_{TD} = 65 m/s

Rješenje:

$$s_L = \frac{{V_{TD}}^2 \cdot m}{2[F_D + \mu(F_G - F_L)]_{sr}}$$

$$V_{sr} = 0.7 \cdot V_{TD} = 0.7 \cdot 65 = 45.5 \quad \text{m/s}$$

$$F_D = C_{D0} \frac{1}{2} \rho {V_{sr}}^2 A = 0.0182 \cdot \frac{1}{2} \cdot 1.225 \cdot 45.5^2 \cdot 511 = 11\,793 \text{ N}$$

$$F_L = 0 \ \ N$$

$$s_L = \frac{65^2 \cdot \frac{2500000}{9.81}}{2[11793 + 0.4 \cdot 2\,500\,000]_{sr}} = 532 \text{ m}$$

8.3. Izračunajte MLM (*Maximum Landing Mass*) prilikom slijetanja zrakoplova Fairchild Republic A-10, ako je potrebna duljina staze do potpunog zaustavljanja nakon dodira sa pistom 250 m (bez reversa potiska). Pretpostaviti da je C_L nakon dodira sa pistom jednak nuli. Maksimalni koeficijent uzgona sa izvučenim zakrilcima u trenutku dodira sa stazom (*touch down*) iznosi 2.8. Aerodrom se nalazi na nadmorskoj visini od 500 m. Podaci:

- A = 47 m^2
- AR = 6.5
- Koeficijent trenja pri kočenju μ = 0.4
- s_L = 250 m
- C_{D0} = 0.032
- C_L = 0
- C_{Lmax}=2.8

Rješenje:

$\rho_{500} = 1.1673$ kg/m^3

$$s_L = \frac{{V_{TD}}^2 \cdot m}{2[F_D + \mu(F_G - F_L)]_{sr}} = \frac{{V_{TD}}^2 \cdot m}{2(F_D + \mu F_G)_{sr}}$$

$$V_{TD} = 1.3 V_{stall} = 1.3\sqrt{\frac{2mg}{\rho A C_{L_{max}}}} \qquad /^2$$

$$V_{TD}^2 = 1.69\frac{2mg}{\rho A C_{L_{max}}} = \frac{3.38mg}{\rho A C_{L_{max}}} = \frac{3.38 \cdot m \cdot 9.81}{1.1673 \cdot 47 \cdot 2.8} = 0.216 \cdot m$$

$$V_{sr} = 0.7 \cdot V_{TD} \quad /^2$$

$$V_{sr}^2 = 0.49 \cdot V_{TD}^2 = 0.49 \cdot 0.216m = 0.106 \cdot m$$

$$F_D = C_{D0}\frac{1}{2}\rho {V_{sr}}^2 A = 0.032 \cdot \frac{1}{2} \cdot 1.1673 \cdot 0.106 \cdot m \cdot 47 = 0.0929 \cdot m \quad \text{N}$$

$$250 = \frac{0.216 \cdot m \cdot m}{2(0.0929 \cdot m + 0.4 \cdot m \cdot 9.81)}$$

$$500(0.0929 \cdot m + 0.4 \cdot m \cdot 9.81) = 0.216m^2$$

$$0.216 \cdot m^2 = 2008.45 \cdot m$$

$$m = 9\,298 \text{ kg}$$

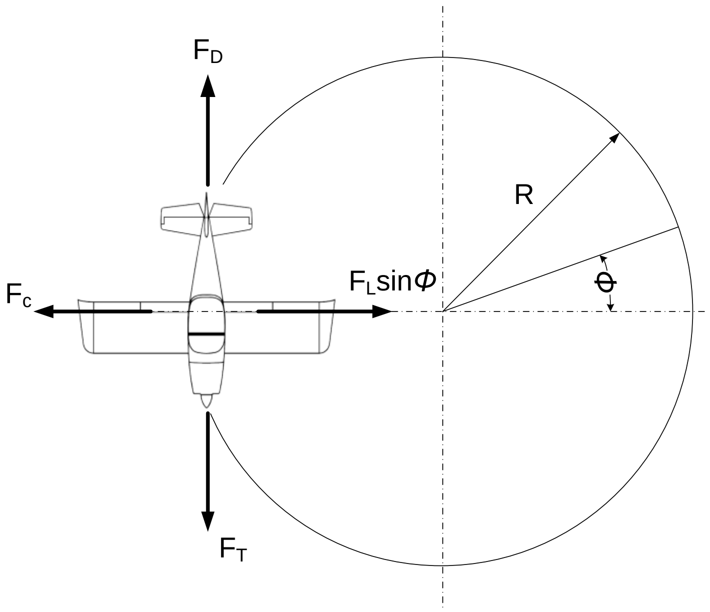
9. HORIZONTALNI I VERTIKALNI ZAOKRET

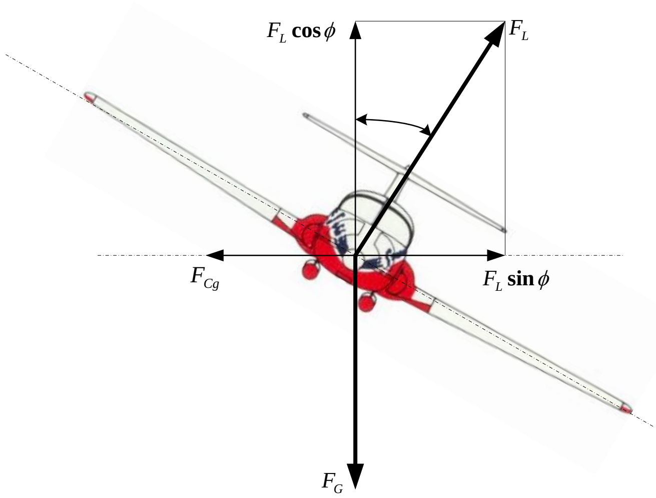
9.1. Maksimalni koeficijent opterećenja za putničke zrakoplove iznosi n_{max} = 2.5. Za zrakoplov koji radi zaokret u horizontalnoj ravnini za 180° pri brzini 120 m/s odredi:

a) minimalni radijus zaokreta

b) kutnu brzinu

c) vrijeme za izvođenje zaokreta





Jednadžbe ravnoteže sila:

$$F_T = F_D$$

$$F_{cp} = F_L \sin\phi$$

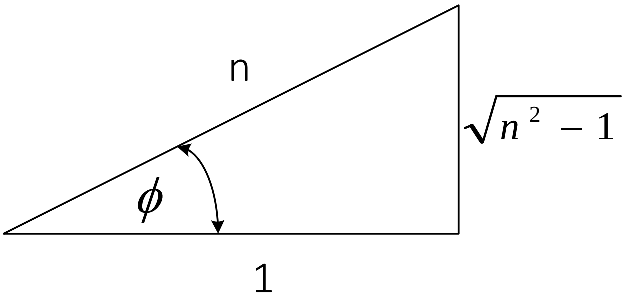
$$F_G = F_L \cos\phi$$

$$F_C = \frac{mV^2}{R} = F_L \sin\phi$$

$$F_L = \frac{F_G}{\cos\phi}$$

$$\frac{mV^2}{R} = \frac{F_G}{\cos\phi}\sin\phi$$

$$tg\phi = \frac{V^2}{gR}$$



$$\frac{F_L}{F_G} = n = \frac{1}{\cos\phi}$$

$$\cos\phi = \frac{1}{n}$$

$$tg\,\phi = \sqrt{n^2 - 1} \qquad \Longrightarrow \quad \frac{V^2}{gR} = \sqrt{n^2 - 1}$$

$$R = \frac{V^2}{g\sqrt{n^2 - 1}} \qquad ;\ R_{min} = \frac{V^2}{g\sqrt{n_{max}^2 - 1}} = \frac{120^2}{9.81\sqrt{2.5^2 - 1}} = 641 \text{ m}$$

$$\omega = \frac{V}{R} = \frac{120}{641} = 0.187 \text{ s}^{-1}$$

$$\omega = \frac{V}{R} = \frac{Vg\sqrt{n^2 - 1}}{V^2} = \frac{g\sqrt{n^2 - 1}}{V}$$

$$\omega = \frac{d\theta}{dt} \qquad\qquad \omega = \frac{\theta}{t} \ \Longrightarrow \ t = \frac{\theta}{\omega} = \frac{\pi}{0.187} = 16.8 \text{ s}$$

$$n = \frac{\omega}{2\pi} = \frac{0.187}{2\pi} = 0.0298 \text{ okr/s}$$

$$n = \frac{\theta}{t} \ \Longrightarrow \ t = \frac{\theta}{n} = \frac{0.5}{0.0298} = 16.8 \text{ s}$$

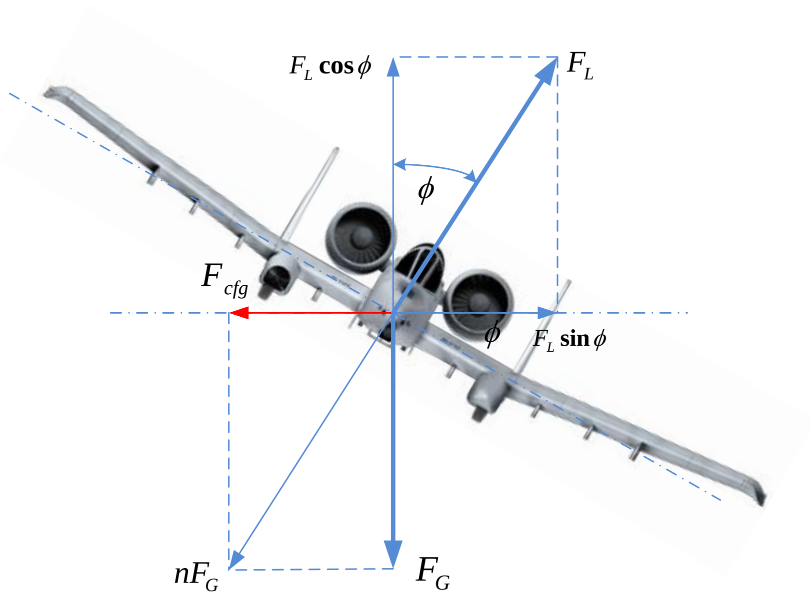
9.2. Za zrakoplov Fairchild Republic A-10 „corner speed“ iznosi 250 kts, a maksimalni koeficijent uzgona bez zakrilaca je 1.2. Odredi minimalni radijus zaokreta i maksimalnu kutnu brzinu na razini mora. Masa zrakoplova je 10500 kg, a površina krila 47 m^2.

Rješenje:

$$F_L = F_G$$

$$\frac{1}{2}\rho V_{min}^2 C_{L_{max}} A = mg$$

$$V_{min} = \sqrt{\frac{2mg}{C_{L_{max}}\rho A}} = \sqrt{\frac{2 \cdot 10500 \cdot 9.81}{1.2 \cdot 1.225 \cdot 47}} = 54.6 \text{ m/s}$$



$$F_G = F_L \cos\phi$$

$$\cos\phi = \frac{F_G}{F_L} = \frac{mg}{C_{L_{max}}\frac{1}{2}\rho V^2 A} = \frac{10500 \cdot 9.81}{1.2 \cdot \frac{1}{2} \cdot 1.225 \cdot 128.6^2 \cdot 47} = 0.1803$$

$$\phi = 79.6\,°$$

$$n_{max} = \frac{F_L}{F_G} = \frac{1}{\cos\phi} = \frac{1}{0.1803} = 5.55$$

$$R_{min} = \frac{V^2}{g\sqrt{n_{max}^2 - 1}} = \frac{128.6^2}{9.81\sqrt{5.55^2 - 1}} = 309 \text{ m}$$

$$\omega_{max} = \frac{V}{R_{min}} = \frac{128.6}{309} = 0.416 \ {}^{\text{rad}}/_{\text{s}} = 23.8 \text{ o/s}$$

9.3. Borbeni zrakoplov ima maksimalni dopušteni koeficijent opterećenja n_{max} = 7. Brzina u horizontalnom zaokretu iznosi 100 m/s. Odredi:

a) minimalni radijus zaokreta

b) kutnu brzinu zaokreta

c) vrijeme potrebno za zaokret

Rješenje:

$$a)\quad R_{min} = \frac{V^2}{g\sqrt{n_{max}^2 - 1}} = \frac{100^2}{9.81\sqrt{7^2 - 1}} = 147.13\ \mathrm{m}$$

$$b)\quad \omega_{max} = \frac{V}{R_{min}} = \frac{100}{147.13} = 0.68\ {}^{\mathrm{rad}}/_{\mathrm{s}} = 38.9\ {}^{\mathrm{o}}/_{\mathrm{s}}$$

$$c)\quad t = \frac{\varphi}{\omega_{max}} = \frac{90}{38.9} = 2.31\ \mathrm{s}$$

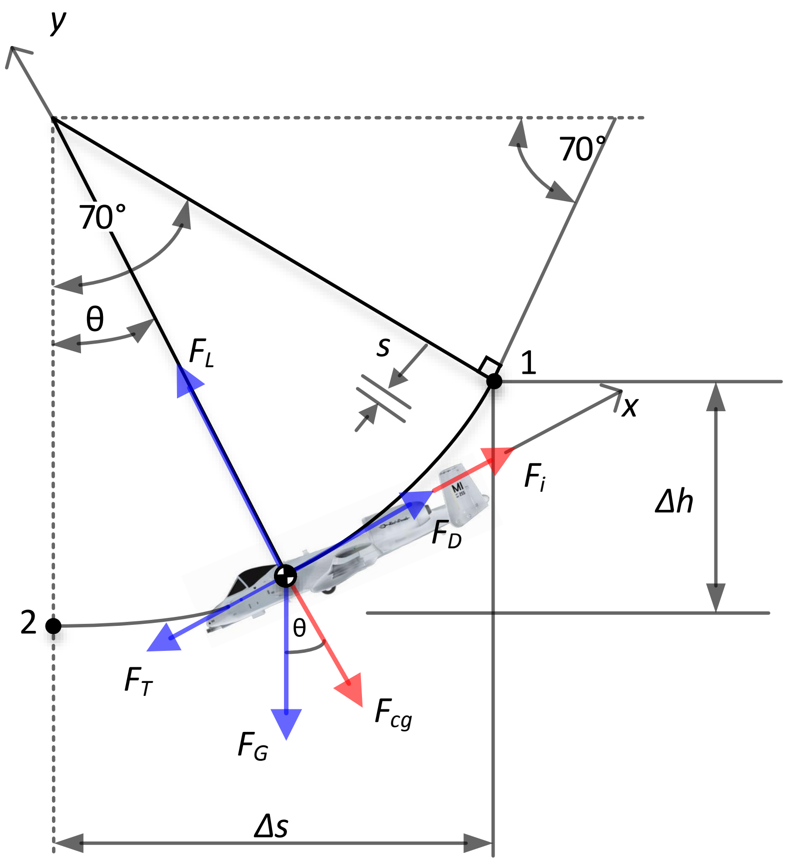
9.4. Borbeni zrakoplov obrušava se pod kutom od 70°. Maksimalni dopušteni faktor opterećenja je pri izvlačenju (točka 2) i iznosi n_{max} = 8. Izvlačenje iz obrušavanja radi se po putanji koja je kružni luk. Brzina ulaska u izvlačenje iznosi 920 km/h.

Odredi:

a) radijus putanje
b) potrebnu visinu za obrušavanje
c) potrebnu duljinu za izvlačenje
d) brzinu na kraju izvlačenja

Rješenje:

$$\boldsymbol{ds = R \cdot d\theta}$$



x: $F_T + F_G sin\theta = F_D + F_i$

y: $F_L = F_C + F_G \cos\theta$

$$F_L = \frac{mV^2}{R} + F_G \cos\theta \quad /: F_G$$

$$\frac{F_L}{F_G} = n = \frac{V^2}{gR} + \cos\theta$$

U točki 1: $\theta_1 = 70°$

$V_1 = 920 \text{ km/h} = 255.6 \text{ m/s}$

$n_1 = ?$

U točki 2: $\theta_2 = 0°$

$V = V_2$

$$n_2 = n_{max} = 1 + \frac{V_2^2}{gR}$$

Pretpostavka: $F_T \approx F_D$

x: $F_G sin\theta = F_i$

y: $F_L = F_C + F_G \cos\theta$

$$x: \quad F_G sin\theta = F_i = m \cdot a = m \cdot \frac{dV}{dt} \qquad (1)$$

$$y: \quad F_L = \frac{mV^2}{R} + F_G \cos\theta \qquad (2)$$

$$\frac{V^2}{R} = \frac{V \cdot V}{R} = \frac{V}{R}\frac{ds}{dt} = \frac{V}{R} \cdot \frac{R(-d\theta)}{dt} = -V\frac{d\theta}{dt}$$

$$ds = R(-d\theta)$$

$$x: \quad m \cdot \frac{dV}{dt} = F_G sin\theta \qquad (1)$$

$$y: \quad -mV\frac{d\theta}{dt} = F_L - F_G \cos\theta \qquad (2)$$

$$(2)/(1) \quad \rightarrow \quad -V\frac{d\theta}{dV} = \frac{F_L - F_G\cos\theta}{F_G sin\theta} \qquad /: F_G$$

$$-V\frac{d\theta}{dV} = \frac{\frac{F_L}{F_G} - \cos\theta}{sin\theta} = \frac{n - \cos\theta}{sin\theta}$$

$$-V\frac{d\theta}{dV} = \frac{n - cos\,\theta}{sin\theta} \qquad /: (-d\theta)$$

$$\frac{V}{dV} = -\frac{n - cos\,\theta}{d\theta sin\theta}$$

$$\frac{dV}{V} = \frac{-sin\theta d\theta}{n - cos\,\theta} = \frac{-sin\theta d\theta}{\frac{V^2}{gR}} / \cdot V^2$$

$$VdV = -gRsin\theta d\theta$$

$$\int_{V_1}^{V_2} VdV = -gR\int_{\theta_1}^{\theta_2} sin\theta d\theta$$

$$\left.\begin{aligned} &\frac{V_2^2 - V_1^2}{2} = -gR(-\cos\theta_2 + \cos\theta_1) \\ &V_2^2 = gR(n_{max} - 1) \end{aligned}\right\}$$

$$gR(n_{max} - 1) - V_1^2 = -2gR(cos\,\theta_1 - cos\,\theta_2)$$

$$gR[(n_{max} - 1) + 2(cos\,\theta_1 - cos\,\theta_2)] = V_1^2$$

$$R = \frac{V_1^2}{g[(n_{max} - 1) + 2(\cos\theta_1 - \cos\theta_2)]} = \frac{\left(\frac{920}{3.6}\right)^2}{9.81[(8-1) + 2(\cos 70° - \cos 0°)]} = 1171 \text{ m}$$

$b)$ $\quad \cos 70° = \dfrac{R - \Delta h}{R}$

$$R\cos 70° = R - \Delta h$$

$$\Delta h = R(1 - \cos 70°) = 1171 \cdot (1 - \cos 70°) = 771 \text{ m}$$

$c)$ $\quad \sin 70° = \dfrac{\Delta s}{R}$

$$\Delta s = R\,\sin 70° = 1171 \cdot \sin 70° = 1100 \text{ m}$$

$d)$ $\quad V_2^2 = gR(n_{max} - 1)$

$$V_2 = \sqrt{gR(n_{max} - 1)} = \sqrt{9.81 \cdot 1171(8-1)} = 284 \text{ m / s} \quad \approx 1021 \text{ km/h}$$

9.5. Zrakoplov radi zaokret na visini 2000 m, konstatnom brzinom od 50 m/s i radijusom od 300 m. Zadane su karakteristike zrakoplova: polara otpora $\boldsymbol{C_D = 0.0323 + 0.104 \cdot C_L^2}$, težina zrakoplova $\boldsymbol{F_G} = 10000\ \mathrm{N}$, površina krila $A = 15\,\mathrm{m}^2$ i učinkovitost elise $\boldsymbol{\eta} = 0.89$. Potrebno je odrediti:

a) normalno opterećenje zrakoplova,

b) kut nagiba zrakoplova,

c) potrebnu silu uzgona i koeficijent uzgona C_L,

d) silu otpora i koeficijent otpora C_D i

e) potrebnu snagu motora (P_{mot}) za taj manevar.

Rješenje:

a) $\frac{V}{R} = \frac{9.81\sqrt{n^2-1}}{V} \quad \Rightarrow n = \sqrt{\left(\frac{V^2}{9.81R}\right)^2 + 1}$

$$n = \sqrt{\left(\frac{V^2}{9.81R}\right)^2 + 1} = \sqrt{\left(\frac{50^2}{9.81 \cdot 300}\right)^2 + 1} = 1.31$$

b) $\phi = \cos^{-1}\left(\frac{1}{n}\right) = \cos^{-1}\left(\frac{1}{1.31}\right) = 40.3°$

c) $F_L = n \cdot F_G = 1.31 \cdot 10000 = 13100\ \mathrm{N}$

$$F_L = \frac{1}{2}\rho V^2 C_L S_{ref}$$

$$C_L = \frac{2F_L}{\rho V^2 S_{ref}} = \frac{2 \cdot 13100}{1.0065 \cdot 50^2 \cdot 15} = 0.694$$

d)

$$C_D = 0.0323 + 0.104 \cdot C_L^2 = 0.0323 + 0.104 \cdot 0.694^2 = 0.0824$$

$$F_D = \frac{1}{2}\rho V^2 C_D S_{ref} = \frac{1}{2} \cdot 1.0065 \cdot 50^2 \cdot 0.0824 \cdot 15 = 1555\ \mathrm{N}$$

e) $P_R = F_D \cdot V = P_{mot} \cdot \eta$

$$P_{mot} = \frac{F_D \cdot V}{\eta} = \frac{1555 \cdot 50}{0.89} = 87.4\ \mathrm{kW}$$

10. UKUPNA ENERGIJA

10.1. Avion leti na 10 000 m brzinom od 60 m/s. Kojom brzinom treba letjeti na visini od 1000 m da bi imao istu energetsku visinu?

$$h_e = h + \frac{V^2}{2g}$$

$$h_{e_{10\,000}} = h_{e_{1\ 000}}$$

$$10000 + \frac{60^2}{2g} = 1000 + \frac{V^2}{2g}$$

$$\frac{V^2}{2g} = 9\,000 + \frac{60^2}{2g}$$

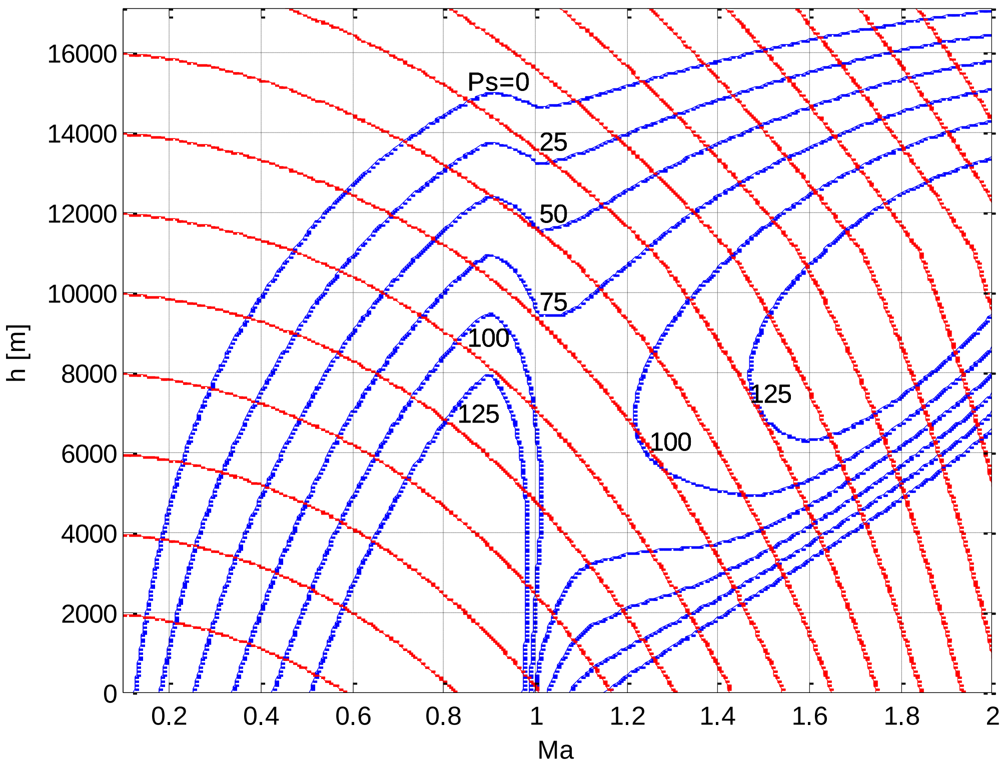
$$V^2 = 18\,000g + 60^2 = 180\,180$$

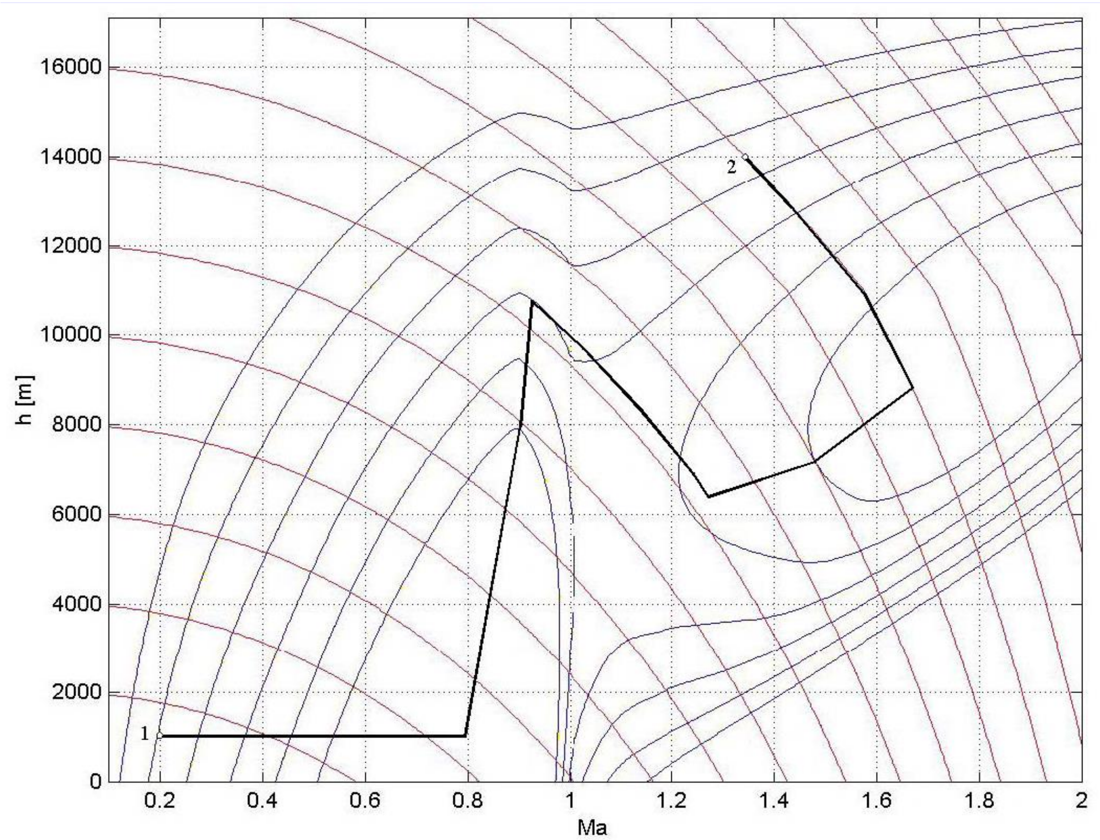
$$V = 424.5 \text{ m/s}$$

Specifična energija = energija po jedinici težine [m]

Energetska visina = geometrijska visina + specifična energija [m]

10.2. Minimalno vrijeme penjanja





Slika 2. Minimalno vrijeme potrebno za penjanje iz pozicije 1 na poziciju 2 (Janković, 2002)

$$\frac{dh_e}{dt} = P_s \quad \Longrightarrow \quad dt = \frac{dh_e}{P_s}$$

$$t = \int_{h_{e1}}^{h_{e2}} \frac{1}{P_s} dh_e$$

$P_s = f(h_e)$ niz točaka koje predstavljaju promjenu režima leta *Ma ,h* od početnog režima leta Ma_1, h_1 u točki 1 u krajnji režim leta Ma_2, h_2 u točki 2.

Da bi vrijeme bilo minimalno, treba režim leta mijenjati tako da vrijednosti funkcije $P_s = f(h_e)$ budu najveće moguće. Režim leta znači određeni Machov broj *Ma* na zadatoj visini *h* .Te dvije vrijednosti predstavljaju određenu točku u dijagramu *Ma-h*. Kroz tu točku prolaze dvije krivulje $h_e(Ma, h) = const$ i $P_S(Ma, h) = const$. Ta točka može biti predstavljena krivolinijskim koordinatama h_e P_S . Zamislimo da mijenjamo tako režim leta da točke na dijagramu sve nalaze na krivulji $h_e(Ma, h) = const$. Vrijednosti P_S od jedne do druge točke će rasti, a zatim opadati zbog oblika krivulja $P_S(Ma, h) = const$, a bit će najveća u onoj točki u kojoj se krivulje $h_e(Ma, h) = const$ i $P_S(Ma, h) = const$ tangiraju. To znači da iz točke koja predstavlja režim leta *1* treba ići u režim leta koji predstavlja točka *2* kroz točke u kojima su krivulje h_e i P_S tangentne jedna na drugu. Ako prijelaz iz režima leta 1 u režim leta 2 vodi pored te zatvorene krivulje, treba režim leta mijenjati duž $h_e = const$da bi se opet išlo kroz točke koje imaju zajedničke tangente za krivulje h_e i P_s. Opravdanje za ovakav postupak jest činjenica da kada mijenjamo režim leta po krivulji $h_e = const$, da je onda $dh_e = 0$, te se vrijednost integrala ne povećava.

$$P_s = \frac{TV - DV}{F_G}$$

10.3. Penjanje s najmanjom potrošnjom goriva

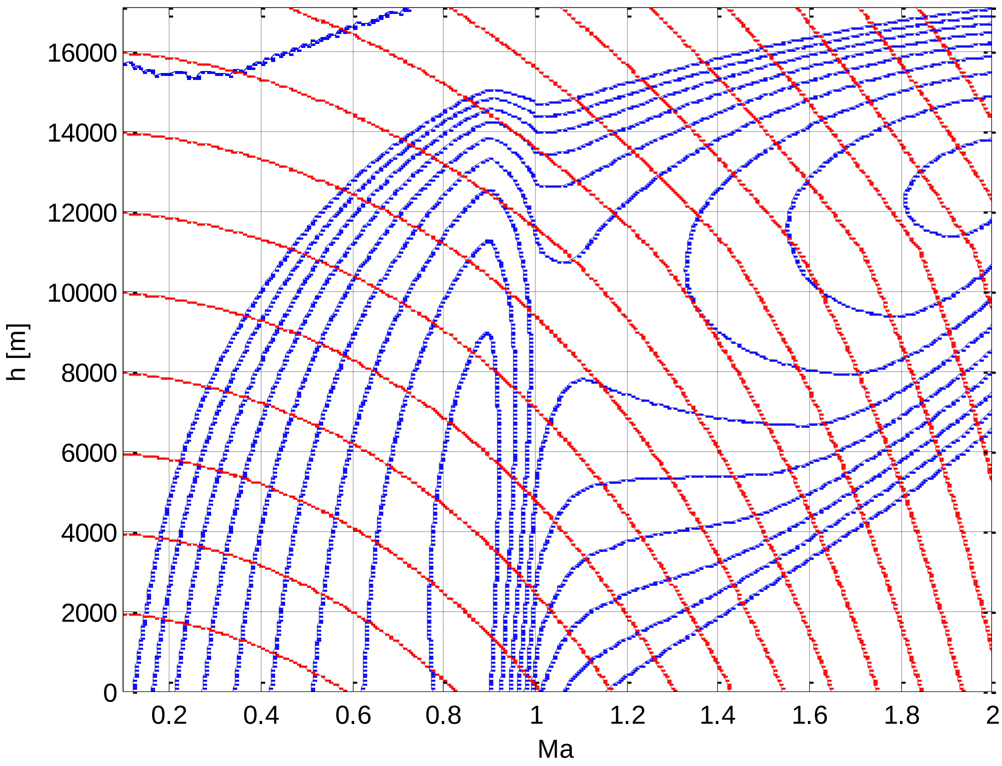
Da bi se odredila promjena režima leta u penjanju tako daje potrošnja goriva najmanja, trebamo funkciju f_s - prirast specifične energije po jedinici goriva (*fuel specific energy*).

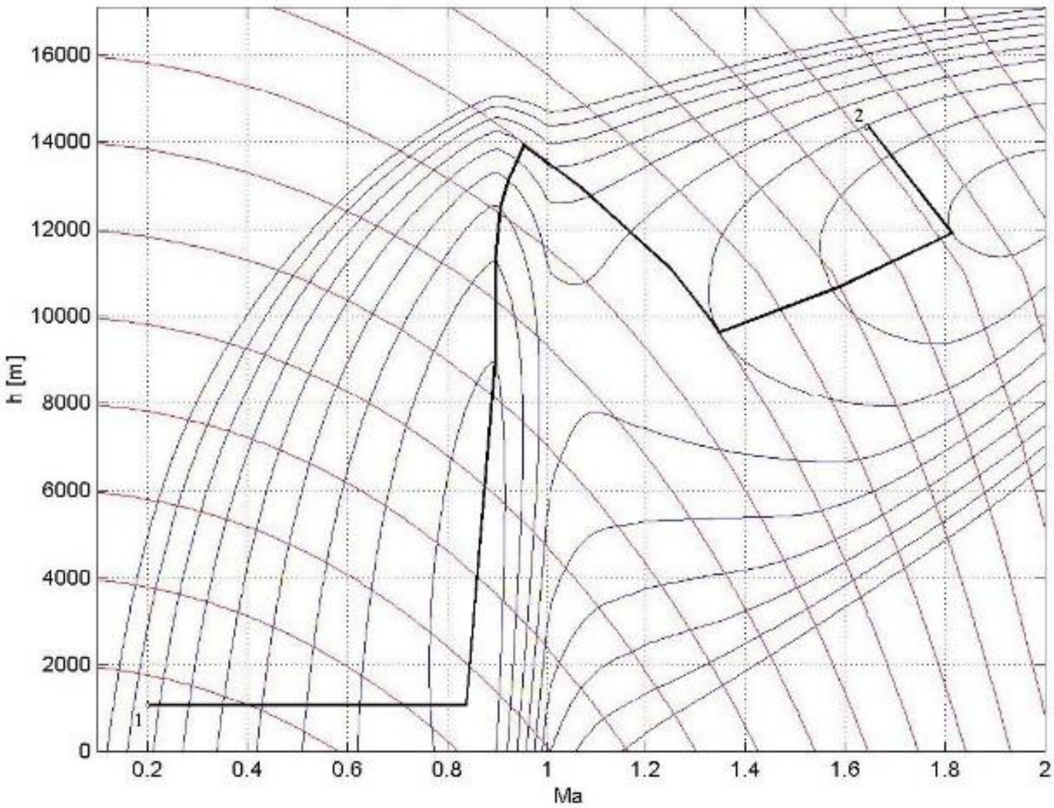
$$\frac{dh_e}{dW_f} = f_s$$

Ako zrakoplov ima mlazni motor za koji je potrošnja goriva proporcionalna pogonskoj sili, onda je

$$f_s = \frac{dh_e}{dW_f} = \frac{P_s dt}{C_T T dt} = \frac{P_s}{C_T T} = f_s(Ma, h, n)$$

$$\Delta W = \int_1^2 \frac{1}{f_s} dh_e$$





Slika 3. Režim penjanja za najmanju potrošnju goriva sa pozicije 1 na poziciju 2 (Janković, 2002).

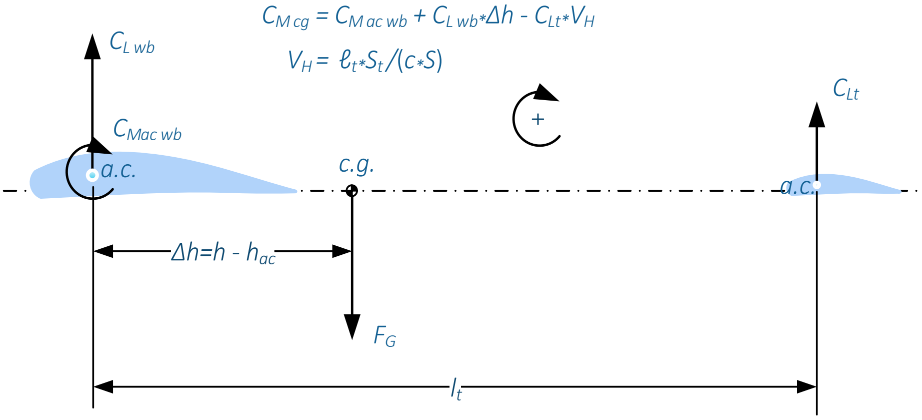
11. UZDUŽNA STABILNOST I UPRAVLJIVOST

Utjecaj pojedinih dijelova zrakoplova na stabilnost je različita i ovisi o položaju aerodinamičkog centra o položaju središta mase. Ispitivanja su pokazala da je u slučaju kombinacije trupa i krila, najčešće položaj aerodinamičkog centra ispred središta mase, te da je $C_{m\,ac,\,wb}$ manji od vrijednosti $C_{m\,ac\,w}$ samoga krila.

Kako je kombinacija krila i tijela zrakoplova nestabilna potrebno je dodati stabilizator koji će stvarati silu s kojom se postiže negativan moment oko središta mase.

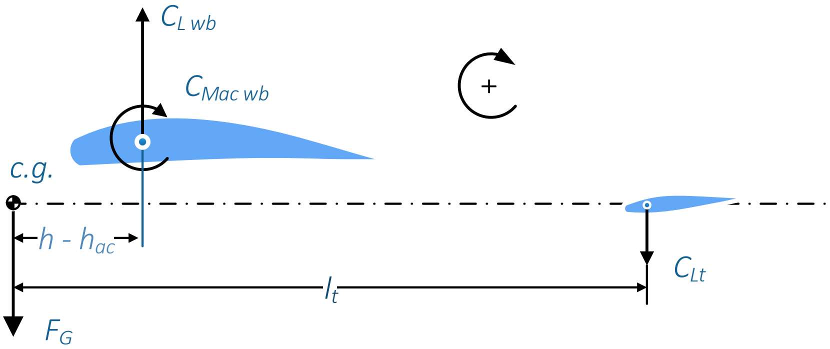
$$C_{M\,cg} = C_{M\,ac\,wb} + C_{L\,wb} * \Delta h - C_{Lt} * V_H$$

$$V_H = \ell_t * S_t / (c * S)$$



Moguća je i suprotna kombinacija, recimo visokokrilci imaju položaj središta mase ispred aerodinamičkog centra kombinacije tijelo-krilo, ali zbog toga moraju na stabilizatoru stvarati dodatno opterećenje prema dolje kako bi se omogućila ravnoteža.

$$C_{M\,cg} = C_{Mac\,wb} - C_{L\,wb} * \Delta h + C_{Lt} * V_H$$



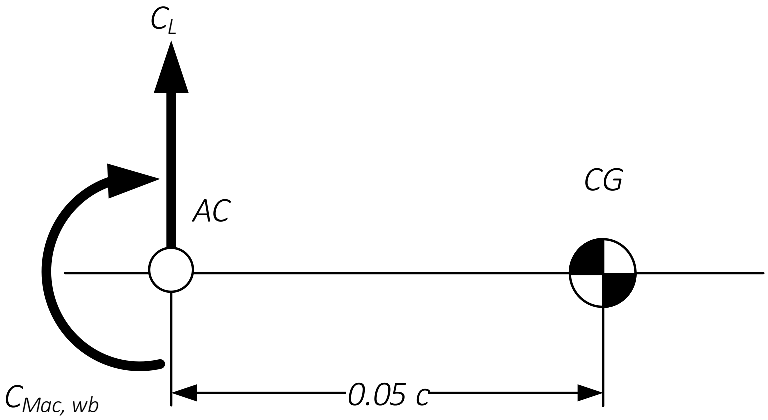
11.1. Za kombinaciju tijelo-krilo, AC je $\mathbf{0.05}c$ ispred CG. Odredite koeficijent momenta propinjanja oko težišta kombinacije.

Zadano:

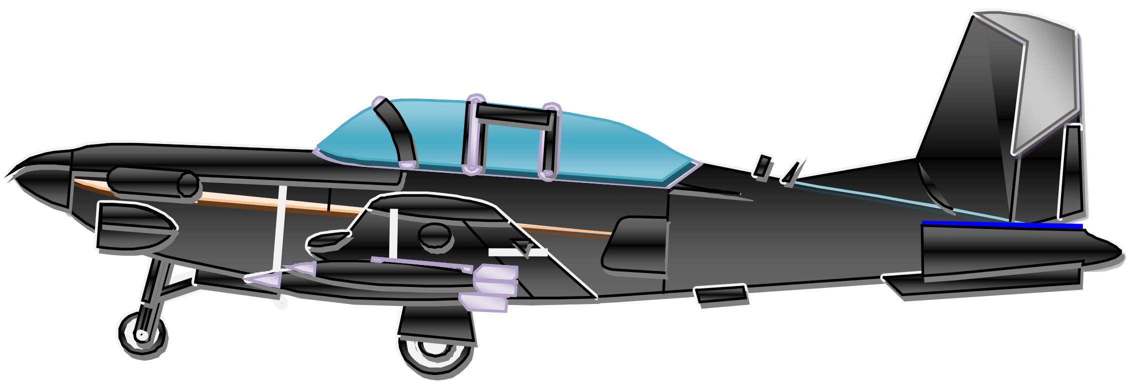
$C_{M_{ac,wb}} = -0.016$

$C_L = 0.45$

Rješenje:



$$C_{M_{cg,wb}} = C_{M_{ac,wb}} + C_{L_{wb}}\left(h - h_{ac_{wb}}\right) = -0.016 + 0.45 \cdot 0.05 = 0.0065$$



11.2. Model tijelo-krilo testiran je u podzvučnom aerotunelu. Uzgon je jednak nuli kada je geometrijski napadni kut $\boldsymbol{\alpha = -1.5°}$. Pri $\boldsymbol{\alpha = 5°, C_L = 0.52}$. Izmjereni su i koeficijenti momenta propinjanja oko težišta:

α [°]	$C_{M\,cg,wb}$
1	-0.01
7.88	0.05

Težište se nalazi na 0.35*c*. Potrebno je odrediti položaj aerodinamičkog centra i vrijednost $C_{M_{ac,wb}}$

Rješenje:

U tablici su pregledno zapisani dobiveni podaci pomoću kojih je potrebno izračunati položaj aerodinamičkog centra i koeficijent momenta propinjanja oko AC.

Tablica 2. $C_{M\,cg,\,wb}$ i $C_{L\,wb}$ pri različitim napadnim kutovima

	$\alpha = 1°$	$\alpha = 7.88°$	$\alpha = -1.5°$	$\alpha = 5°$
$C_{M_{cg,wb}}$	-0.01	0.05		
$C_{L_{wb}}$			0	0.52

$$C_{L_{wb}} = a_{wb}\alpha_{wb}$$

$$a_{wb} = \frac{dC_L}{d\alpha} = \frac{C_{L2} - C_{L1}}{\alpha_2 - \alpha_1} = \frac{0.52 - 0}{5 + 1.5} = 0.08 \quad / \;°$$

ili

$\alpha_{z0} = -1.5° \;\Longrightarrow\; za\;\; C_L = 0.52,\;\; \alpha_a = \alpha - \alpha_{z0} = 5 - \;(-1.5) = 6.5°$ pa slijedi da je

$$a_{wb} = \frac{C_L}{\alpha_a} = \frac{0.52}{6.5} = 0.08 \quad / \;°$$

$$C_{M_{cg,wb}} = C_{M_{ac,wb}} + a_{wb}\alpha_{a_{wb}}\left(h - h_{ac_{wb}}\right)$$

$$\begin{aligned} \alpha = 1° \quad &\rightarrow \quad -0.01 = C_{M_{ac,wb}} + 0.08 \cdot (1 + 1.5)\left(h - h_{ac_{wb}}\right) \\ \alpha = 7.88° \quad &\rightarrow \quad 0.05 = C_{M_{ac,wb}} + 0.08 \cdot (7.88 + 1.5)\left(h - h_{ac_{wb}}\right) \quad /- \end{aligned}$$

$$0.06 = 0 + 0.08 \cdot 6.88\left(h - h_{ac_{wb}}\right)$$

$h - h_{ac_{wb}} = 0.11$ aerodinamički centar kombinacije nalazi se 11% c ispred CG, odnosno

$h_{ac_{wb}} = 0.35 - 0.11 = 0.24$ gdje *h* znači od početka aerodinamičke tetive (24% c mjereno od početka SAT)

$$-0.01 = C_{M_{ac,wb}} + 0.08 \cdot (1 + 1.5) \cdot 0.11$$

$$C_{M_{ac,wb}} = -0.032$$

11.3. Promatra se model tijelo-krilo iz prethodnog primjera 11.2:

$\boldsymbol{C_{M_{ac,wb}} = -0.032}$

$\boldsymbol{h - h_{ac_{wb}} = 0.11}$

$\boldsymbol{a_{wb} = 0.08}$ / °

$\boldsymbol{\alpha_{L0} = -1.5}$°

Površina krila je S = 0.1 m^2, a dužina tetive krila c = 0.1 m. Pretpostavimo da je modelu tijelo-krilo dodan horizontalni rep. Udaljenost težišta zrakoplova od AC repa je $\boldsymbol{\ell_t = 0.17\ \mathrm{m}}$, površina repa $\boldsymbol{S_t}$ = 0.03 m^2, a postavni kut repa je 2.7°. Nagib krivulje koeficijenta uzgona repa je $\boldsymbol{a_t}$ = 0.1 po stupnju, a iz eksperimentalnih mjerenja poznate su vrijednosti kuta skretanja struje zraka na repu $\boldsymbol{\varepsilon_0 = 0}$° i i njegova promjena po napadnom kutu $\boldsymbol{d\varepsilon/d\alpha = 0.35}$. Ako je napadni kut modela $\boldsymbol{\alpha = 7.88}$°, odrediti koeficijent momenta oko težišta $\boldsymbol{C_{M_{cg}}}$ modela zrakoplova.

Rješenje:

$$C_{M_{cg}} = C_{M_{ac,wb}} + a \cdot \alpha_a \left[h - h_{ac_{wb}} - V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right] + V_H a_t (i_t + \varepsilon_0)$$

$$\alpha_a = 7.88 + 1.5 = 9.38°$$

$$V_H = \frac{\ell_t \cdot S_t}{c \cdot S} = \frac{0.17 \cdot 0.02}{0.1 \cdot 0.1} = 0.34$$

$$a_t = 0.1 \quad po\ °$$

$$\frac{\partial \varepsilon}{\partial \alpha} = 0.35$$

$$i_t = 2.7°$$

$$\varepsilon_0 = 0$$

$$C_{M_{cg}} = -0.032 + 0.08 \cdot 9.38 \left[0.11 - 0.34 \frac{0.1}{0.08}(1 - 0.35)\right] + 0.34 \cdot 0.1(2.7 + 0) = -0.065$$

11.4. Je li model iz primjera 11.3 uzdužno uravnotežen i statički stabilan?

Rješenje:

Nužni uvjeti za uzdužnu ravnotežu i statičku stabilnost su:

1° $\frac{\partial C_{M\,cg}}{\partial \alpha_a} < 0$

2° $C_{M\,0} > 0$

3° α_e ...mora biti u rasponu mogućih napadnih kutova u letu za taj zrakoplov

$$C_{M\,cg} = C_{M\,ac,wb} + a \cdot \alpha_a \left[h - h_{ac_{wb}} - V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right] + V_H a_t (i_t + \varepsilon_0)$$

$$\frac{\partial C_{M\,cg}}{\partial \alpha_a} = a\left[h - h_{ac_{wb}} - V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right]$$

$$C_{M\,0} = \left(C_{M\,cg}\right)_{L=0} = C_{M\,ac,wb} + V_H a_t (i_t + \varepsilon_0)$$

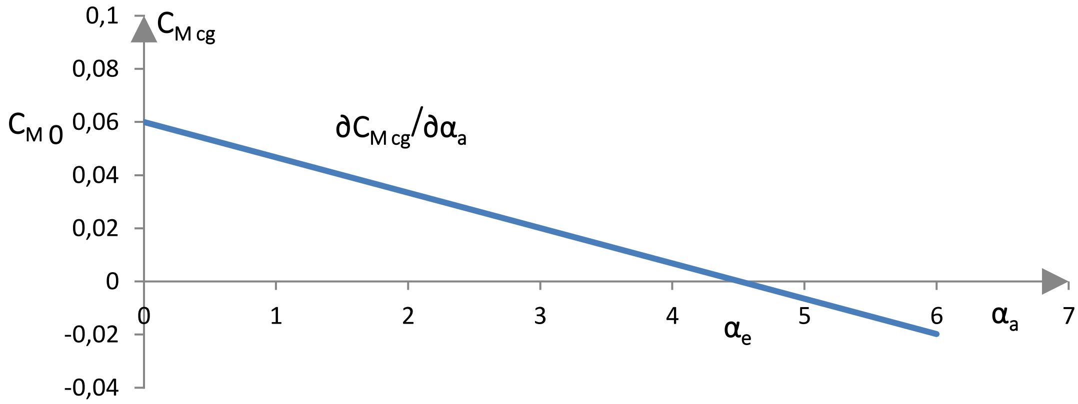
1° $\frac{\partial C_{M\,cg}}{\partial \alpha_a} = 0.08\left[0.11 - 0.34 \frac{0.1}{0.08}(1 - 0.35)\right] = -0.0133 \quad < 0 \qquad \surd$

2° $C_{M\,0} = -0.032 + 0.34 \cdot 0.1(2.7 + 0) = 0.06 \quad > 0 \qquad \surd$

$$C_{M\,cg} = \frac{\partial C_{M\,cg}}{\partial \alpha_a} \cdot \alpha_a + C_{M\,0}$$

$$0 = -0.0133\alpha_e + 0.06 \qquad \Rightarrow \alpha_e = \frac{0.06}{0.0133} = 4.5°$$

3° $\alpha_e = 4.5° \qquad \surd$



11.5. Model krilo-tijelo testiran je u zračnom tunelu. Uvjeti strujanja u test sekciji jednaki su ISA/SL s brzinom od 100 m/s. Površina krila S = 1.5 m^2, a dužina tetive $\boldsymbol{c} = \mathbf{0.45\ m}$. Uz pomoć mjernih instrumenata tunela izmjeren je moment oko težišta kada je uzgon jednak nuli M_0 = -12.4 Nm. Kada se promijenio napadni kut modela izmjereni su moment oko težišta M_{cg} = 20.67 Nm i uzgon F_L = 3675 N. Odrediti:

a) $\boldsymbol{C_{M_{ac}}}$ i položaj aerodinamičkog centra AC modela?

b) Modelu je dodana masa na zadnji dio, te se njegovo težište pomaknulo prema natrag za 20% tetive. Koliki je u tom slučaju moment $\boldsymbol{M_{cg_{wb}}}$ ako je izmjerena sila uzgona od $\boldsymbol{F_{L_{wb}}} = \mathbf{4\ 000\ N}$?

Rješenje:

a)

$$M_{cg_{wb}} = M_{ac_{wb}} + F_{L_{wb}}(h - h_{ac_{wb}})c \qquad (1)$$

$F_{L\,wb}$ = 0 N	M_0 = -12.4 Nm	$-12.4 = M_{ac_{wb}}$ (1)	$M_{ac_{wb}} = -12.4$ Nm
$F_{L\,wb}$ = 3675 N	$M_{cg_{wb}}$ = 20.67 Nm	$20.67 = -12.4 + 3675(h - h_{ac_{wb}}) \cdot 0.45$ (1)	$h - h_{ac_{wb}} = 0.02$

$$C_{M_{ac,wb}} = \frac{M_{ac_{wb}}}{\frac{1}{2}\rho_\infty V^2 S c} = \frac{-12.4}{0.5 \cdot 1.225 \cdot 100^2 \cdot 1.5 \cdot 0.45} = -0.0034$$

b)

$h = 0.02 + h_{ac_{wb}}$ … položaj težišta bez dodane mase

$h = 0.02 + h_{ac_{wb}} + 0.2$ … položaj težišta sa dodanom masom

$$h - h_{ac_{wb}} = 0.02 + 0.2 = 0.22$$

$$M_{cg_{wb}} = M_{ac_{wb}} + F_{L_{wb}}(h - h_{ac_{wb}})c = -12.4 + 4000 \cdot 0.22 \cdot 0.45 = 383.6\ \text{Nm}$$

11.6. Za model zrakoplova za kojega su poznati slijedeći podaci:

$\boldsymbol{V_H = 0.34}$

$\boldsymbol{C_{M_{ac}} = -0.032}$

$\boldsymbol{h_{ac_{wb}} = 0.24}$

$\boldsymbol{h = 0.35}$

$\boldsymbol{a = 0.08}$ / °

S = 19 m²

F_G = 22 700 N

$\boldsymbol{a_t = 0.1 \ po}$ °

$\boldsymbol{\dfrac{\partial \varepsilon}{\partial \alpha} = 0.35}$

$\boldsymbol{i_t = 2.7°}$

$\boldsymbol{\varepsilon_0 = 0}$

$\boldsymbol{\dfrac{\partial C_{L_t}}{\partial \delta_e} = 0.04}$

Odredite:

a) položaj neutralne točke, h_n

b) statičku marginu, $h_n - h$

c) potreban otklon elevatora za uravnoteživanje zrakoplova (δ_{trim}) pri brzini od 61 m/s na SL

Rješenje:

a)

$$h_n \equiv h \quad \Rightarrow \quad \frac{\partial C_{M_{cg}}}{\partial \alpha_a} = 0$$

$$\frac{\partial C_{M_{cg}}}{\partial \alpha_a} = a\left[h - h_{ac_{wb}} - V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right] = 0$$

$$h_n - h_{ac_{wb}} - V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) = 0$$

$$h_n = h_{ac_{wb}} + V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)$$

$$h_n = 0.24 + 0.34 \frac{0.1}{0.08}(1 - 0.35) = 0.516$$

$h_n > h$ ... zrakoplov je statički stabilan

b)

$h_n - h = 0.516 - 0.35 = 0.166$ ... statička margina (što veća, zrakoplov je stabilniji)

Provjera:

$$\frac{\partial C_{M_{cg}}}{\partial \alpha_a} = a\left[h - h_{ac_{wb}} - V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right] \qquad (1)$$

$$h_{ac_{wb}} = h_n - V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) \qquad (2)$$

$$(2) \rightarrow (1) \quad \Longrightarrow \frac{\partial C_{M\,cg}}{\partial \alpha_a} = a\left[h - h_n + V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) - V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right] = a(h - h_n)$$

$$\frac{\partial C_{M\,cg}}{\partial \alpha_a} = -a(h_n - h) = -0.08 \cdot 0.166 = -0.0133$$

c)

$$\delta_{trim} = \frac{C_{M\,0} + \dfrac{\partial C_{M\,cg}}{\partial \alpha_a} \alpha_n}{V_H \dfrac{\partial C_{L\,t}}{\partial \delta_e}}$$

$$C_{M\,0} = \left(C_{M\,cg}\right)_{L=0} = C_{M\,ac} + V_H a_t (i_t + \varepsilon_0) = -0.032 + 0.34 \cdot 0.1 \cdot (2.7 + 0) = 0.06$$

$$F_G = F_L$$

$$F_G = C_L \frac{1}{2} \rho V^2 S$$

$$C_L = \frac{2F_G}{\rho V^2 S} = \frac{2 \cdot 22\,700}{1.225 \cdot 61^2 \cdot 19} = 0.52$$

$$a = \frac{C_L}{\alpha_a} \quad \Longrightarrow \quad \alpha_a = \frac{C_L}{a} = \frac{0.52}{0.08} = 6.5° \; \equiv \; \alpha_n$$

$$\delta_{trim} = \frac{0.06 - 0.0133 \cdot 6.5}{0.34 \cdot 0.04} = -1.94°$$

Kako bi se zrakoplov uravnotežio na napadnom kutu od 6.5°, elevator se mora zakrenuti za 1.94° prema gore.

11.7. Odredite *stick-free* (puštena palica[2]) statičku stabilnost zrakoplova (isti kao u zadatku 11.6) za kojega su poznate slijedeće aerodinamičke karakteristike i derivativi njegovog zglobnog momenta elevatora:

$\frac{\partial C_{h_e}}{\partial \alpha_t} = -0.008$ ...promjena koeficijenta zglobnog momenta elevatora po α_t

$\frac{\partial C_{h_t}}{\partial \delta_e} = -0.013$ ...promjena koeficijenta zglobnog momenta stabilizatora po δ_e

$\frac{\partial C_{L_t}}{\partial \delta_e} = 0.04$ ...promjena koeficijenta uzgona po δ_e

$C_M{}'_0 = ?$

$h'_n = ?$

$\frac{\partial C_M{}'_{cg}}{\partial \alpha_a} = ?$

Poznato:

$V_H = 0.34$ $\quad a = 0.08\ po\ °$

$C_{M_{ac}} = -0.032$ $\quad a_t = 0.1\ \ po\ °$

$h_{ac_{wb}} = 0.24$ $\quad i_t = 2.7°$

$h = 0.35$ $\quad \varepsilon_0 = 0$

$\frac{\partial \varepsilon}{\partial \alpha} = 0.35$

Rješenje:

Faktor slobodnog elevatora obilježava se velikim slovom *F*[3].

$$F = 1 - \frac{1}{a_t}\frac{\partial C_{L_t}}{\partial \delta_e} \cdot \frac{\partial C_{h_e}/\partial \alpha_t}{\partial C_{h_t}/\partial \delta_e} = 1 - \frac{1}{0.1} \cdot 0.04 \cdot \frac{-0.008}{-0.013} = 0.754$$

$$C_M{}'_0 = C_{M_{ac}} + F \cdot V_H a_t (i_t + \varepsilon_0) = -0.032 + 0.754 \cdot 0.34 \cdot 0.1 \cdot (2.7 + 0) = 0.037$$

$$h'_n = h_{ac} + F \cdot V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) = 0.24 + 0.754 \cdot 0.34 \frac{0.1}{0.08}(1 - 0.35) = 0.448$$

$h'_n - h = 0.448 - 0.35 = 0.098$ statička margina za *stick – free*

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[2] Slučaj kada je elevator slobodan rotirati oko zgloba pod utjecajem struje zraka koja djeluje na njega. U takvom slučaju je manja stabilnost nego za fiksne upravljačke površine. Slučaj puštene palice i fiksne predstavljaju dva ekstrema.

[3] *Free elevator factor* – faktor slobodnog elevatora uvijek je manji od jedan.

Ako se usporedi položaj neutralne točke sa $h_n = 0.516$ vidi se da se neutralna točka pomaknula prema naprijed čime je smanjena stabilnost (i povećana upravljivost).

$h_n - h = 0.166$ ... statička margina (što veća, zrakoplov stabilniji)

$\frac{0.166 - 0.098}{0.166} = 0.41$ ... 41% je smanjena statička stabilnost

$$\frac{\partial {C_M}'_{cg}}{\partial \alpha_a} = -a({h'}_n - h) = -0.08 \cdot 0.098 = -0.0078$$

PRILOZI

A. Karakteristike zrakoplova Cessna Skylane i Cessna Citation 3

AVION	Cessna Skylane 	Cessna Citation 3 
b [m]	10.9	16.2
A [m²]	16.2	29.5
m [kg]	1338	8988
C_{D0}	0.025	0.02
e	0.8	0.81
$AR = \frac{b^2}{A}$	7.3	8.9
Pogon	jednomotorni elisno -klipni	dvomotorni turbomlazni
P_{mot} [kW]	171.5	-
F_T [N]	-	32472
η	0.8	-
c	0.27 kg/kWh	$1.667 \cdot 10^{-4}$ 1/s

B. Tablica standardne atmosfere

H [m]	T [K]	p [Pa]	ρ [kg/m^3]	a [m/s]	ν [m^2/s]
0	288,15	101325	1,225	340,3	1,460E-05
500	284,9	95460,1	1,1673	338,4	1,519E-05
1000	281,65	89873,2	1,1116	336,4	1,582E-05
1500	278,4	84554,1	1,0580	334,5	1,647E-05
2000	275,15	79492,7	1,0065	332,5	1,716E-05
2500	271,9	74679,6	0,9568	330,6	1,789E-05
3000	268,65	70105,2	0,9091	328,6	1,866E-05
3500	265,4	65760,4	0,8632	326,6	1,947E-05
4000	262,15	61636,2	0,8191	324,6	2,033E-05
4500	258,9	57724,1	0,7767	322,6	2,123E-05
5000	255,65	54015,4	0,7361	320,5	2,219E-05
5500	252,4	50502,1	0,6971	318,5	2,321E-05
6000	249,15	47176,2	0,6596	316,4	2,428E-05
6500	245,9	44029,9	0,6238	314,4	2,542E-05
7000	242,65	41055,7	0,5894	312,3	2,663E-05
7500	239,4	38246,4	0,5566	310,2	2,792E-05
8000	236,15	35594,7	0,5251	308,1	2,929E-05
8500	232,9	33094	0,4950	305,9	3,074E-05
9000	229,65	30737,4	0,4663	303,8	3,229E-05
9500	226,4	28518,6	0,4388	301,6	3,394E-05
10000	223,15	26431,3	0,4126	299,5	3,570E-05
10500	219,9	24469,5	0,3877	297,3	3,758E-05
11000	216,65	22627,3	0,3639	295,1	3,958E-05
11500	216,65	20916	0,3363	295,1	4,282E-05
12000	216,65	19330,1	0,3108	295,1	4,634E-05
13000	216,65	16509,9	0,2655	295,1	5,425E-05
14000	216,65	14101,2	0,2267	295,1	6,352E-05
15000	216,65	12044,0	0,1937	295,1	7,437E-05
16000	216,65	10286,8	0,1654	295,1	8,707E-05
17000	216,65	8786,0	0,1413	295,1	1,019E-04
18000	216,65	7504	0,1207	295,1	1,194E-04
19000	216,65	6409,4	0,1031	295,1	1,397E-04
20000	216,65	5474,3	0,0880	295,1	1,636E-04

C. Popis formula

Horizontalni let

polara aviona: $C_D = C_{D0} + C_{Di} = C_{D0} + \frac{C_L^2}{\pi \cdot e \cdot AR}$; uzgon: $F_L = C_L \frac{1}{2}\rho V^2 A$; otpor: $F_D = C_D \frac{1}{2}\rho V^2 A$

opće jednadžbe gibanja aviona: $F_T \cdot \cos\alpha_T - F_D - F_G \cdot \sin\theta = m\frac{\mathrm{d}V}{\mathrm{d}t}$; $F_L + F_T \cdot \sin\alpha_T - F_G \cdot \cos\theta = m\frac{V^2}{r}$

jednadžbe gibanja za hor. let pravocrtnom putanjom stalnom brzinom: $F_T = F_D$; $F_L = F_G$

aspektni odnos: $AR = \frac{b^2}{A}$; brzina leta: $V = \sqrt{\frac{2F_G}{C_L \rho A}}$; minimalna brzina: $V_{\min} = \sqrt{\frac{2F_G}{C_{L\max}\rho A}}$

potrebni potisak: $F_{TR} = F_D = F_G \frac{C_D}{C_L}$; $F_{TR} = \frac{1}{2}C_{D0}\rho A V^2 + \frac{2F_G^2}{\pi e AR \rho A}\cdot\frac{1}{V^2}$; opterećenje krila: $\frac{F_G}{A}$

minimalni potrebni potisak, najbolji dolet: $C_{D0} = C_{Di} = \frac{C_L^2}{\pi \cdot e \cdot AR}$; finesa aviona: $f = \left(\frac{C_L}{C_D}\right)_{\max}$

potrebna snaga: $P_R = F_{TR}V$; $P_R = \frac{C_D}{C_L^{1,5}}\sqrt{\frac{2F_G^3}{\rho A}}$; $P_R = \frac{1}{2}C_{D0}\rho A V^3 + \frac{2F_G^2}{\pi e AR \rho A}\cdot\frac{1}{V}$

min. potrebna snaga, najveća istrajnost leta: $P_{R\min} = \left(\frac{C_D}{C_L^{1,5}}\right)_{\min}\sqrt{\frac{2F_G^3}{\rho A}}$; $C_{D0} = \frac{1}{3}C_{Di} = \frac{C_L^2}{3\pi \cdot e \cdot AR}$

raspoloživa (korisna) snaga: $P_A = F_{TA}V$; $P_A = \eta P$; (η – stupanj korisnog djelovanja propelera)

maksimalna brzina: $P_A = P_R$; utjecaj visine: $V_h = V_0\sqrt{\frac{\rho_0}{\rho_h}}$; $P_{Rh} = P_{R0}\sqrt{\frac{\rho_0}{\rho_h}}$; $T_{TRh} = F_{TR0}$

Breguetove formule - propelerski pogon: $R = \frac{\eta}{g \cdot c}\frac{C_L}{C_D}\ln\frac{m_0}{m_1}$; $E = \frac{\eta}{c \cdot g^{1,5}}\frac{C_L^{1,5}}{C_D}\sqrt{2\rho A}\left(\frac{1}{\sqrt{m_1}} - \frac{1}{\sqrt{m_0}}\right)$

dolet i istrajnost - mlazni pogon: $R = \frac{1}{c_t}\frac{C_L^{0,5}}{C_D}\sqrt{\frac{8}{\rho A}}\left(\sqrt{F_{G0}} - \sqrt{F_{G1}}\right)$; $E = \frac{1}{c_t}\frac{C_L}{C_D}\ln\frac{m_0}{m_1}$

specifična potrošnja goriva: $c = SFC\left[\frac{\mathrm{kg}}{\mathrm{Ws}}\right]$; $c_t = TSFC\left[\frac{\mathrm{N}}{\mathrm{Ns}} = \frac{1}{\mathrm{s}}\right]$

Penjanje

brzina uzdizanja: $R/C = V\sin\theta$; $R/C = \frac{P_A - P_R}{F_G}$; $(R/C)_{\max} = \frac{(P_A - P_R)_{\max}}{F_G}$

kut penjanja: $\sin\theta = \frac{R/C}{V}$; $\theta = \arcsin\left(\frac{R/C}{V}\right)$; $\theta_{\max} = \arcsin\left(\frac{R/C}{V}\right)_{\max}$

vrijeme penjanja: $t = \int_{h_1}^{h_2}\frac{\mathrm{d}h}{R/C}$; $t = \frac{h_2 - h_1}{(R/C)_{sr}} = \frac{\Delta h}{(R/C)_{sr}}$; $(R/C)_{sr} = \frac{(R/C)_1 + (R/C)_2}{2}$

apsolutni vrhunac (plafon) leta: $R/C = 0$; praktični vrhunac (plafon) leta: $R/C = 0{,}5\,\mathrm{m/s}$

Spuštanje

brzina propadanja: $R/D = V\sin\theta$; $R/D = \left(\frac{C_D}{C_L^{1,5}}\right)\sqrt{\frac{2F_G}{\rho A}}$

kut spuštanja: $\tan\theta = \frac{F_D}{F_L} = \frac{C_D}{C_L}$; $\tan\theta_{\min} = \left(\frac{C_D}{C_L}\right)_{\min} = \frac{1}{(C_L/C_D)_{\max}}$

Uzlijetanje

duljina zaleta: $s_{LO} = \frac{V_{LO}^2 \cdot m}{2\{F_T - [F_D + \mu(F_G - F_L)]\}_{sr}} = \frac{S^2 \cdot F_G^2}{g\rho A C_{L\max}\{F_T - [F_D + \mu(F_G - F_L)]\}_{sr}}$

S - faktor sigurnosti; μ - koeficijent trenja; brzine: $V_{stall} = \sqrt{\frac{2F_G}{C_{L\max}\rho A}}$; $V_{LO} = SV_{stall}$; $V_{sr} = 0{,}7V_{LO}$

uzgon: $F_L = C_L \frac{1}{2}\rho V_{sr}^2 A$; otpor: $F_D = (C_{D0} + \phi C_{Di})\frac{1}{2}\rho V_{sr}^2 A$; utjecaj blizine tla: $\phi = \frac{(16h/b)^2}{1+(16h/b)^2}$

Slijetanje

duljina kočenja: $s_L = \frac{V_T^2 \cdot m}{2[F_D + \mu(F_G - F_L)]_{sr}} = \frac{S^2 \cdot F_G^2}{g\rho A C_{L\max}[F_D + \mu(F_G - F_L)]_{sr}}$

duljina kočenja uz revers potiska: $s_L = \frac{S^2 \cdot F_G^2}{g\rho A C_{L\max}\{F_{TR} + [F_D + \mu(F_G - F_L)]\}_{sr}}$

S - faktor sigurnosti; μ - koeficijent kočenja; brzine: $V_{stall} = \sqrt{\frac{2F_G}{C_{L\max}\rho A}}$; $V_T = SV_{stall}$; $V_{sr} = 0{,}7V_T$

uzgon: $F_L = C_L \frac{1}{2}\rho V_{sr}^2 A$; otpor: $F_D = (C_{D0} + \phi C_{Di})\frac{1}{2}\rho V_{sr}^2 A$; utjecaj blizine tla: $\phi = \frac{(16h/b)^2}{1+(16h/b)^2}$

Zaokreti

- u horizontalnoj ravnini:

radijus zaokreta: $R = \frac{V^2}{g\sqrt{n^2-1}}$; $R_{\min} = \frac{V^2}{g\sqrt{n_{\max}^2-1}}$; kutna brzina: $\varpi = \frac{V}{R} = \frac{g\sqrt{n^2-1}}{V}$

faktor opterećenja: $n = \frac{F_L}{F_G}$; $n = \frac{1}{\cos\phi}$; centrifugalna sila: $F_c = \frac{mV^2}{R}$

- pull-up manevar: $R = \frac{V^2}{g(n-1)}$; $\varpi = \frac{g(n-1)}{V}$

- pull-down manevar: $R = \frac{V^2}{g(n+1)}$; $\varpi = \frac{g(n+1)}{V}$

<u>Stabilnost i upravljivost aviona</u>

položaj težišta: $x_{CG} = \dfrac{\sum M_i}{\sum m_i}$; $M_i = m_i \cdot x_i$; x_i - krak (udaljenost) hvatišta težine do referentne crte

moment oko težišta: $M_{CG} = C_{MCG} \dfrac{1}{2} \rho V^2 Ac$; apsolutni napadni kut: $\alpha_a = \alpha - \alpha_{L0}$; $(\alpha_{L0} < 0)$

koef. momenta oko težišta: $C_{MCG} = C_{MAC} + C_L \left[\dfrac{x_{CG}}{c} - \dfrac{x_{AC}}{c} \right] - V_H C_{LH}$

$C_L = a \cdot \alpha_a$; $C_{LH} = a_H \cdot \alpha_H$, $\alpha_H = \alpha_a - i_H - \varepsilon$; $\varepsilon = \varepsilon_0 + \dfrac{\partial \varepsilon}{\partial \alpha} \alpha_a$; volumen. omjer repa: $V_H = \dfrac{l_H \cdot A_H}{c \cdot A}$

uvjeti uzdužne statičke stabilnosti aviona: $C_{M0} > 0$ i $\dfrac{\partial C_{MCG}}{\partial \alpha_a} < 0$

$C_{M0} = C_{MAC} + V_H a_H (i_H + \varepsilon_0)$; $\dfrac{\partial C_{MCG}}{\partial \alpha_a} = a \left[\dfrac{x_{CG}}{c} - \dfrac{x_{AC}}{c} - V_H \dfrac{a_H}{a} \left(1 - \dfrac{\partial \varepsilon}{\partial \alpha} \right) \right]$

neutralna točka: $\dfrac{\partial C_{MCG}}{\partial \alpha_a} = 0$; $\left(\dfrac{x}{c} \right)_n = \dfrac{x_{AC}}{c} + V_H \dfrac{a_H}{a} \left(1 - \dfrac{\partial \varepsilon}{\partial \alpha} \right)$; statička rezerva: $\left(\dfrac{x}{c} \right)_n - \left(\dfrac{x}{c} \right) = -\dfrac{1}{a} \dfrac{\partial C_{MCG}}{\partial \alpha_a}$

utjecaj otklona elevatora na uzgon horizontalnog repa: $C_{LH} = \dfrac{\partial C_{LH}}{\partial \alpha_H} \alpha_H + \dfrac{\partial C_{LH}}{\partial \delta_e} \delta_e = a_H \alpha_H + \dfrac{\partial C_{LH}}{\partial \delta_e} \delta_e$

$\dfrac{\partial C_{LH}}{\partial \delta_e}$ - djelotvornost elevatora; otklon elevatora za $C_{MCG} = 0$: $\delta_{etrim} = \dfrac{C_{M0} + (\partial C_{MCG} / \partial \alpha_a) \cdot \alpha_a}{V_H (\partial C_{LH} / \partial \delta_e)}$

BIBLIOGRAFIJA

[1] Anderson, J. D. (1999.). *Aircraft Performance and Design.* New York: McGraw-Hill.

[2] Anderson, J. (2001). *Fundamentals of Aerodynamics.* New York: McGraw-Hill.

[3] Anderson, J. (2000). *Introduction to Flight.* New York: McGraw Hill.

[4] Denker, J. S. (2008). *See How It Flies.* Preuzeto 15. 04 2013 iz See How It Flies: http://www.av8n.com/how/

[5] Dunn, D. J. (n.d.). *Kinematics - Velocity and Acceleration diagrams.* Preuzeto 2015 iz Freestudy: http://www.freestudy.co.uk/d225/t7.pdf

[6] Janković, S. (2002). *Mehanika leta zrakoplova.* Zagreb: Fakultet strojarstva i brodogradnje.

[7] Kesić, P. (2003). *Osnove aerodinamike.* Zagreb: FSB.

[8] McCormick, B. (1995). *Aerodynamics, Aeronautics and Flight Mechanics.* New York: John Wiley & Sons.

[9] Rendulić, Z. (1984). *Aerodinamika.* Zemun: RO Sava Mihić.